Math 5616, Spring 2000 **Assignments** are due at the start of class on the given day

Assignment 6, Book Problems: Due Apr 24

Chapter 7, # 11; Chapter 8, # 2, 5, 7, 9, 10.

Special Problem 5: Due Apr 3

(a) Prove that, if $f(x)$ is defined for $x \geq 0$, continuous at 0, differentiable for $x > 0$, and $f'(x) \to L$ as $x \downarrow 0$ then f is differentiable form the right at 0 and its right-hand derivative there has value L.

(b) Prove that the function $f(x) := e^{-1/x}$ for $x > 0$ and $f(x) \equiv 0$ for $x \leq 0$ has continuous derivatives of all orders. Construct a function $F(x)$ on $\mathbb{R}$ that is not identically zero, has continuous derivatives of all orders, and that *is* identically zero outside $(-1, 1)$. Although Taylor's *THEOREM* works well at -1 and at 1, the Taylor *SERIES* at each of these points are identically zero, hence do not represent the function in a nbd of each of those two points.

Assignment 5, Book Problems: Due FRIDAY March 24

Chapter 7, # 14, 18-20, 24.

Special Problem 4: Due Mar 15

Chapter 7, # 25.

Assignment 4, Book Problems: Due Mar 13

Chapter 7, # 2, 3, 12, 13 (12 & 13 count as $1\frac{1}{2}$ problems each).

Special Problem 3: Due Feb 25: 20 points

Suppose that $f : [a, b] \to \mathbb{R}$.
(a) Prove that, for all $c \in (a, b)$, $Q_f[a, b] = Q_f[a, c] + Q_f[c, b]$, where Q is any one of V, P or N.
(b) It is true, and difficult to prove, that if f is continuous and of bounded variation on $[a, b]$, then

$$(*) \qquad V_f[a, b] = \lim_{\text{mesh}(\pi) \to 0} V_f(\pi).$$

Clarify the meaning of $\lim_{\text{mesh}(\pi) \to 0}$ and construct an example of a function f of bounded variation on $[a, b]$ such that $(*)$ is false. Hint: Consider some variation on the ruler function whose values form an absolutely convergent series.
(c) Prove that, if f is Lipschitz continuous, then f is of bounded variation on $[a, b]$ and $(*)$ holds.
Hint for $(*)$: Get close with some partition. Use the short-interval device. Use common refinements and make an observation about their variations that resembles facts about upper and lower sums.

Assignment 3, Book Problems: Due Feb 23

Chapter 6, # 9, 11, 12, 13 (counts as 2 problems).

Assignment 2, Book Problems: Due Feb 11

Chapter 6, # 7–9, 16(a); Prove 6.13, with $\alpha(x) = x$.

Special Problem 2: Due Feb 16

(a) Prove that, if Z_n is a set of measure zero, for $n \in \mathbb{Z}^+$, then $Z := \bigcup_{n=1}^{\infty} Z_n$ is also a set of measure zero.
(b) Construct an open set $\Omega \subseteq \mathbb{R}$, the sum of whose lengths of intervals is less than $\epsilon < 1$, such that Ω contains all the rational numbers in $[0, 1]$. Show that the characteristic function of Ω, namely $f(x) := [x \in \Omega]$, is not equal a.e. to any function that is continuous a.e. **Note:** The Dirichlet function *is* equal a.e. to a function that is continuous a.e., for example, the zero function, the ruler function. Hence functions that are continuous a.e. and zero a.e. can be lumped togethe, via an equivalence relation, to play the rôle of "zero."

Assignment 1, Book Problems: Due Jan 31

Chapter 5, # 26 (uses MVT); Chapter 6, # 2, 4, 5, 8.

Special Problem 1: Due Feb 7

Chapter 6, # 10, with $\alpha(x) = x$.