Math 8602, Spring 2002 Assignments

Assignments are due at the start of class on the due date. Further Problems may be scored competitively! Give credit for help received, including books and hints from me and others; mention discussions. If a Further Problem is difficult, please include a “narrative” that tells what you went thru to reach your results.

Further Problem 5
Due May 8

(a) Show that the set $SM(X, \Sigma)$ of signed measures defined on (X, Σ) is normed space with respect to $\|\varphi\| := V(X)$.
(b) Prove *(iv)* of Theorem 10.33.
(c) Is $(SM, \|\cdot\|)$ a Banach space? Remark: The Theorems of Fubini and Tonelli apply to double sums!

Assignment 13
Due May 6: Book Problems

Chapter 10, # 24; Chapter 11, # 2, 3, 5.

Assignment 12
Due Apr 29: Book Problems

Chapter 10, # 2, 9, 12, 23.

Assignment 11
Due Apr 22: Book Problems

Chapter 9, # 13; Chapter 10, # 10, 16, 18 (take what they say about Hahn–Banach for granted).

Assignment 10
Due Apr 15: Book Problems

Chapter 9, # 4, 8, 9.

Further Problem 4
Due Apr 10

Solve Chapter 9, # 3, with *uniform* continuity. In addition show that, if $1 < p < \infty$, then $f * K \to 0$ at infinity. Show also that this need not be true if $p = 1$ or $p = \infty$.

Assignment 9
Due Apr 8: Book Problems

Chapter 8, # 16, 20, 21, Chapter 9, # 2.

Further Problem 3
Due Apr 3

(a) Prove that the span of a set $S \subseteq H$ is dense in the Hilbert space H if and only if $y \perp S \Rightarrow y = 0$.
(b) Is (a) true in ineer product spaces?
(c) Use (a) to prove that the simple functions in $L^2(E)$ are dense in $L^2(E)$.

Assignment 8
Due Apr 1: Book Problems

Chapter 8, # 14b, 15, 17, 22.

Assignment 7
Due Mar 25: Book Problems

Chapter 8, # 7, 10, 11, 14a.

Assignment 6
Due Mar 11: Book Problems

Chapter 8, # 2: second sentence, 3: just (8.13), 6, 8.

Assignment 5
Due Mar 4: Book Problems

Chapter 8, # 2: first sentence, 3: just (8.12), 4, 5.

Further Problem 2
Due Mar 6

Suppose that $\varphi : (a, b) \to \mathbb{R}$.

(a) [3] Suppose also that for all x, y and z in (a, b), if $x < y < z$ then $\dfrac{\varphi(y) - \varphi(x)}{y - x} \leq \dfrac{\varphi(z) - \varphi(y)}{z - y}$.
Prove that φ is convex on (a, b).

(b) [7] Suppose now (instead of as in (a)) that the graph of φ has a supporting line at each of its points.
Prove that φ is convex on (a, b).

Assignment 4
Due Feb 25: Book Problems

Chapter 7, # 10, 12, 15; Chapter 8, # 1.

Assignment 3
Due Feb 18: Book Problems

Chapter 7, # 6 − 9, 17.

Assignment 2
Due Feb 11: Book Problems

Chapter 7, # 1, 2(as is), 5, 16, “18:” Compute the Hardy-Littlewood maximal function of the characteristic function of the interval $[-1, 1] \subseteq \mathbb{R}$.

Assignment 1
Due Feb 4: Book Problems

Chapter 6, # 6, 10; Chapter 7, # 2(only for f that are continuous and have compact support, and with “Legesgue set” replaced by “in $\mathbb{R}^n$”), 3, 4.

Further Problem 1
Due Feb 6

Let $S = \{s_k\}_{k=1}^{\infty}$ be an arbitrary denumerable subset of $\mathbb{R}^n$. For example, S could be the set of points in $\mathbb{R}^n$ all of whose coordinates are rational. We define $F(E) := \sum_{s_k \in E} 2^{-k}$. Show that F is a “set function” as defined on page 98. Which sigma–algebras can be used? Is F “continuous?” Why? Define “continuous near x_o” and show that F is continuous near x_o if $x_o \notin S$.