Math 8802, Spring 2006 **Assignments** are due at the start of class on the Due date

Assignments are due at the start of class on the due date. Further Problems may be scored competitively! Neatness, legibility and cogency count! Give credit for help received, including books and hints from me and others; mention discussions. If a Further Problem is difficult, please include a "narrative" that tells what you went thru to reach your results. Further Problems should be written on standard size paper, and spiral-bound paper must be trimmed! There should be one-inch margins all around (name and problem number in the top margin are OK). Lined paper is fine, and using both sides is fine too. But if your handwriting fills up lines, double-space!

Take-Home Final: Due May 10, by 10:30am

(1) Chapter 7, # 11; **(2)** Chapter 7, # 16; **(3)** (cf 12.22(b)) Show that, if $f : H \to H$ is an onto *function* and $\langle f(x), f(y)\rangle = \langle x, y\rangle$ for all x and y in H, then $f(x) = Ux$, where U is a unitary operator in $B(H)$;
(4) Find the resolution of the identity for the Fourier transform on $H := L^2(\mathbb{R}^d)$ (suitably normalized to make it unitary) also see Chapter 12, # 23.

Assignment 2, Book Problems: Due Apr 5

Chapter 6, # 4, 6a, 7, 9ab, 12, 20; Chapter 7, # 1, 3, 4, 7, 8, 14.

Further Problem 3: Due Mar 22

Prove or disprove: $\mathcal{S}(\mathbb{R}^d)$ is dense in $\mathcal{S}'(\mathbb{R}^d)$. The topology to use for $\mathcal{S}'(\mathbb{R}^d)$ is its usual topology, the weak-star topology.

Further Problem 2: Due Feb 20

Suppose that H is an infinite-dimensional complex Hilbert space, and that $\mathcal{O}$ is an orthonormal basis.
Let $t : \mathcal{O} \to \mathbb{C}$ and suppose that for $y \in \mathcal{O}$, $t_y \to 0$ "at infinity" (clarify). Prove or disprove that

$$Tx := \sum_{y\in\mathcal{O}} t_y \langle x, y\rangle\, y \quad \text{is compact and (in either case) find } \sigma(T).$$

Assignment 1, Book Problems: Due Feb 13

Chapter 4, # 3, 4, 6, 9, 10, 12, 13, 17, 27.

Further Problem 1: Due Feb 6

Prove or disprove: A continuous linear mapping T between Banach spaces X and Y is invertible if and only if T^* is invertible.