

Indicate your approach! Show your work! **Justify** your answers! Good Luck! There are 5 pages, and 100 points.

1 (10) Define **linearly independent set**. Is the set $\{(3, 2, 6), (2, 1, -1), (2, 1, -2)\}$ linearly independent?

2 (10) Define **open set**. Is the set of all real numbers that are not integers an open subset of \mathbb{R} ?

3 (10) Define **boundary point**. Which points in the plane (\mathbb{R}^2) are boundary points for the set of all (x, y) with $y < x$?

4 (10) Define **differentiable** for a function of the type $f : \mathbb{R}^n \rightarrow \mathbb{R}$. Is the function $f(x, y)$ defined for non-zero (x, y) by $\frac{xy}{x^2 + y^2}$, with $f(0, 0) = 0$ differentiable at $(0, 0)$? Differentiable elsewhere?

5 (10) Find the square of $A := \begin{pmatrix} 3 & 2 & 2 \\ 2 & 1 & 1 \\ 6 & -1 & -2 \end{pmatrix}$.

6 (10) Let $f(x, y, z) := \begin{pmatrix} y + z \\ x + z \\ x + y \end{pmatrix}$. Let $g := f \circ f$. Find $g(x, y, z)$ and $g'(x, y, z)$.

7 (10) Let \mathbb{C} denote the set of all matrices of the form $\begin{pmatrix} x & -y \\ y & x \end{pmatrix}$, where x and y are real numbers. Show that arbitrary pairs of matrices in \mathbb{C} commute, and that the two columns of all but one of the matrices in \mathbb{C} are linearly independent.

8 (10) Let $f(x, y) = e^{x-y}$. Find the directional derivative of f , in the increasing- t direction of the curve $g(t) = (t^3, t^2)$, at $(-1, 1)$.

9 (10) Find the gradient of $f(x, y, z) := \sqrt{x^2 + 2y^2 + 3z^2}$.

10 (10) Define **differentiable** for functions of the type $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$. If f is differentiable at $x_o \in \mathbb{R}^n$, what are the entries in the derivative? Find the derivative of $f(x, y) = \begin{pmatrix} x^3 - xy^2 \\ x^2y - y^3 \end{pmatrix}$.