

Indicate your approach! Show your work! **Justify** your answers! Good Luck! There are 5 pages, and 100 points.

1 (10) Define **critical point**. Find the critical points of $f(x, y) = x^2 - 4xy + y^2 - 8x + 16y$

2 (10) Define **extreme value**. Find the extreme values of in the domain $x^2 + y^2 + z^2 \leq 4$.

3 (10) Define **reduced row echelon form (rref)**. Reduce $A := \begin{pmatrix} 3 & 2 & 2 \\ 2 & 1 & 1 \\ 6 & -1 & -2 \end{pmatrix}$ to reduced row echelon form.

Check your work!

4 (10) Define **kernel** for a matrix of size $m \times n$, and find a basis for the kernel of $B := \begin{pmatrix} 3 & 2 & 3 \\ 2 & 1 & 1 \\ 2 & 3 & 7 \end{pmatrix}$.

5 (10) Define **image** for a matrix of size $m \times n$, and find a basis for the image of $B = \begin{pmatrix} 3 & 2 & 3 \\ 2 & 1 & 1 \\ 2 & 3 & 7 \end{pmatrix}$.

6 (10) Define **linear transformation**. Let $f(x, y, z) := \begin{pmatrix} y + z \\ x + z \\ x + y \end{pmatrix}$. Let $g := f \circ f$. Show that g is linear and find its matrix.

7 (10) Find A , B and C such that $\frac{x^2 + 2x + 3}{(x - 1)(x - 2)(x - 3)} = \frac{A}{x - 1} + \frac{B}{x - 2} + \frac{C}{x - 3}$.

8 (10) Find all solutions of $\begin{cases} 3x + 5y + 3z = 5 \\ 7x + 9y + 19z = 2 \end{cases}$

9 (10) Define **orthonormal set**. Find a vector that is orthogonal to each of $\begin{pmatrix} 1 \\ 2 \\ 3 \end{pmatrix}$ and $\begin{pmatrix} 4 \\ 5 \\ 6 \end{pmatrix}$, and make the three vectors into an orthonormal set.

10 (10) Define **differentiable** for functions of the type $f : \mathbb{R}^n \rightarrow \mathbb{R}^m$. If f is differentiable at $x_0 \in \mathbb{R}^n$, what are the entries in the derivative? Find the derivative of $f(x, y) = \begin{pmatrix} x^3 - xy^2 \\ x^2y - y^3 \end{pmatrix}$.

You might find it useful to work #19, p71; #39, p72; #32, p85(then replace 3×2 by 2×3 and do it again); #33, p86; #46, p87; #50abc, p87; #31-35, p107; #23, #24, p131; #11, #12, p143.

Under what conditions on the vectors a and b in \mathbb{R}^n is $I_n + ab^T$ invertible?