

Indicate your approach! Show your work! **Justify** your answers! Good Luck! There are 5 pages, and 100 points.

1 (10) Define **determinant**. Find $\det(A)$, where $A := \begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$.

2 (10) Find the **QR decomposition** of the matrix in # 1. **Briefly** describe what Q and R are and how they are found.

3 (10) Define **projection**. Find $\text{proj}_{\text{Im}(B)}$, where $B := \begin{pmatrix} 1 & -2 & 1 \\ -3 & 6 & -3 \\ 3 & 6 & -3 \\ -1 & 2 & 1 \end{pmatrix}$.

4 (10) Define **orthogonal matrix**. Show that the columns of an $n \times n$ orthogonal matrix form a basis for \mathbb{R}^n .

5 (10) Use Gauss–Jordan elimination methods to find $\det \begin{pmatrix} 1 & -2 & 1 & 2 \\ -3 & 6 & -3 & 3 \\ 3 & 6 & -3 & 6 \\ 1 & 2 & 1 & 2 \end{pmatrix}$.

6 (10) Find the minimum distance from the solution–set of $3x + 6y + 7z + 5w = 3$ and $2x + 4y + 5z + 4w = -1$ to the origin.

7 (10) Find an orthonormal basis for the kernel of the matrix $C := \begin{pmatrix} 1 & -2 & 1 \\ -3 & 6 & -3 \\ -3 & 6 & -3 \\ -1 & 2 & -1 \end{pmatrix}$.

8 (10) Suppose the columns of an $n \times n$ matrix D are orthogonal but not orthonormal. Show that D is invertible, and say how to find the inverse of D .

9 (10) Suppose $\begin{pmatrix} A & B \\ C & D \end{pmatrix}$ consists of square matrices (all the same size).

How are $\det \begin{pmatrix} A & B \\ C & D \end{pmatrix}$ and $\det \begin{pmatrix} B & A \\ D & C \end{pmatrix}$ related to each other?

10 (10) Find a cubic polynomial whose graph comes as close as possible to the points the points $(-1, 0)$, $(0, 1)$, $(1, 0)$, $(2, 1)$ and $(3, 2)$.

More later...

Hint for # 6: drop a perpendicular... the solution-set is parallel to a certain subspace...

Hint for # 10: Use least–squares...

11 (10) State the **Cauchy–Buniakowsky–Schwarz Inequality**. Show that if vectors x and y in \mathbb{R}^n satisfy $|x + y|^2 = |x|^2 + |y|^2$ then $x \perp y$.