Ask! Indicate your approach! Show your work! Good Luck! There are 5 pages, and 100 points.

(1) [10] State the Schwarz Inequality for vectors in $\mathbb{R}^k$ and apply it to show that

$$d(p,\, q) := \sqrt{|p_1 - q_1|^2 + \cdots + |p_k - q_k|^2}$$

satisfies the triangle inequality.

(2) [10] Give a *self-contained* proof that there is no rational number whose square is 5.

(3) [10] Prove that the union of two denumerable sets is denumerable.

(4) [15] Define *uncountable.* Prove that the set of all sequences whose terms all belong to the set $\{0, 1, 2\}$ is uncountable.

Scratch Page **Be sure to CLEARLY link work here to a problem! Put the link THERE too!**

(5) [15] Prove that if $f(x)$ is increasing and 0 at 0 and $f(x+y) \leq f(x) + f(y)$ for all non-negative x and y, then $f(d(p,\,q))$ is a metric on a set X whenever $d(p,\,q)$ is a metric on X.

(6) [10] State and prove the factorization of the difference of n-th powers.

(7) [10] State the Corollary that relates the n-th root of a product to the product of the n-th roots, and explain how it follows from the Theorem of which it is a Corollary.

(8) [10] Define *open set* and *closed set* in a metric space. Use Rudin's definitions. **Briefly** explain why, if a set E is closed, its complement, E^c, is open.

(9) [10] Define *metric space.* Show that $\mathbb{R}^2$ is a metric space with respect to $d(p, q) := |p_1 - q_1| + |p_2 - q_2|$. Here, $p = (p_1, p_2)$, and so on.