Ask! Indicate your approach! Show your work! Good Luck! There are 10 problems, 6 pages, and 130 points.

(1) [10] State the Principle of Mathematical Induction, and prove by induction that

$$\text{for all } (n \in \mathbb{N})(2^n > n).$$

(2) [15] State and prove the Difference-of-Powers Formula.

(3) [15] Prove that the equation $r^2 = 5$ has no solution $r \in \mathbb{Q}$

(4) [10] Prove that, if A and B are subsets of a set X, then:

$$A \subseteq B \text{ if and only if } A \cap B = A \text{ if and only if } A \cup B = B.$$

(5) [10] Express the denial of $P \vee Q \vee R$ as a conjunction of denials. Similarly, if A, B and C are subsets of a set X, express the complement of $A \cup B \cup C$ as an intersection of complements. Explain, **briefly.**

(6) [20] Construct a wide truth table that: (a) shows $A \Rightarrow B$ and its contrapositive are logically equivalent but that $A \Rightarrow B$ and its converse are not; (b) shows whether or not, if C denotes the converse of $A \Rightarrow B$, the statement $(A \Rightarrow B) \vee C$ is a tautology.

Scratch Page **Be sure to CLEARLY link work here to a problem! Put the link THERE too!**

(7) [10] Show that $5^n - 1$ is a multiple of 4, for all $n \in \mathbb{N}$.

(8) [10] Prove that for all $n \in \mathbb{N}$, $1 + 2^3 + \cdots + n^3 \left[= \sum_{m=1}^{n} m^3 \right] = \left(\frac{n(n+1)}{2} \right)^2$.

(9) [10] Write the following statement "in logic," using logic symbols and our symbolic notation for the set names:

"Between any two distinct real numbers there is an integer."

Explain why the statement is false.

(10) [20] State the Completeness Axiom. Prove that, if S is a non-empty set of real numbers and r is a real number such that

(a) r is an upper bound for S

and (b) $(\forall x \in \mathbb{R})[x < r \Rightarrow (\exists s \in S)(s > x)]$,

then $r = \sup S$.