Math 2582H, Fall '02, **Some integral problems**

In each case, find

$$\iint_T dA, \quad \iint_T x\,dA \quad \text{and} \quad \iint_T y\,dA,$$

and thus the centroid of T, when T is

- The triangle with vertices $(0, 0)$, $(1, 0)$ and $(\frac{1}{2}, \frac{\sqrt{3}}{2})$.
- The triangle with vertices $(0, 0)$, $(1, 0)$ and $(0, R)$ (with $R > 0$).
- The triangle with vertices $(-1, -2)$, $(3, 1)$ and $(0, 4)$.
- The part of the unit circle "to the right of" a line thru the origin.
- The part of the unit circle "to the right of" a line *not* thru the origin.
- A circular disk tangent to, and to the right of the y-axis.

Try to find a way to do each integral that minimizes work.

Use Cavalieri's Principle to find

- The volume of a regular tetrahedron.
- The volume of a pyramid whose base is a contented set S whose area is A.
- The volume of a "bowl," bounded above by $z = b$, below by $z = a$ and bounded "on the sides" by $z = r^2$. Here, $0 < a < b$.
- The volume obtained by rotating the region under the graph of a positive continuous function, defined on $[a, b]$, about the x-axis.
- The volume obtained by rotating the graph of a positive continuous function defined on $[a, b]$ about the line $y = -c$, where $c > 0$.

Try to find a way to do each integral that minimizes work.

Use Pappus's Theorem to find the volume obtained by rotating

- The triangle with vertices $(0, 0)$, $(1, 0)$ and $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ about the x-axis.
- The triangle with vertices $(0, 0)$, $(1, 0)$ and $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ about the y-axis.
- The triangle with vertices $(0, 0)$, $(1, 0)$ and $(\frac{1}{2}, \frac{\sqrt{3}}{2})$ about each of its edges.

Do the same for each of the other 4 T's you found the centroid of.

- The volume of a "bowl," bounded above by $z = b$, below by $z = a$ and bounded "on the sides" by $z = r^2$. Here, $0 < a < b$.
- The volume obtained by rotating the region under the graph of a positive continuous function, defined on $[a, b]$, about the x-axis.
- The volume obtained by rotating the graph of a positive continuous function defined on $[a, b]$ about the line $y = -c$, where $c > 0$.

Try to find a way to do each integral that minimizes work.

- Find the volume of a ball that has had a hole drilled thru it (along a diameter).
- Find the volume of a spherical cap (the part of a ball of radius R that lies "beyond" a plane at a distance $0 < r < R$ from the origin).
- Find the volume of a spherical "cone," (the part of a ball of radius R that lies within a right circular cone with its vertex at the origin).