

This question was asked about problem 14 in Chapter 3: is there a measurable set of infinite measure with  $|E|_i \neq \infty$ ?

Answer: No.

*Proof:* Let  $|E| = \infty$ . For each  $k \in \mathbb{Z}^+$  we set  $E_k := E \cap B_k(0)$ . Then, for  $k$  so large that  $|E_k| > 1$ , by Lemma (3.22) (the characterization of measurability in terms of closed sets), there exists a closed set  $F_k \subseteq E_k$  such that  $|E_k \setminus F_k| < 1/k$ . Thus (for review, why?)  $|E_k| - 1/k < |F_k| \leq |E|_i$ . It follows that  $|E|_i = \infty$ .