Ask! Indicate your approach! Show your work!

Quiz 4: July 25.

(1) [10] Prove that, if $f(x)$ and $g(x)$ are both differentiable in (a, b) and $f'(x) = g'(x)$ for all $x \in (a, b)$, then $f(x)$ and $g(x)$ differ by a constant.

(2) [15] Suppose that $f''(x) := \frac{d}{dx} f'(x)$ exists for all x in (a, b), and that $f''(x) = C$ (a constant) for all x in (a, b). Prove that $f(x)$ is a polynomial of degree at most 3, and find the coefficient of x^3 (the constant that multiplies x^3 in the polynomial formula).

(3) [20] Suppose that $f(x)$ is differentiable in (a, b) and that for some $x_o \in (a, b)$, $f''(x_o)$ exists (see # 2 for Definition of f''). Prove that

$$\lim_{h \to 0} \frac{f(x_o + h) + f(x_o - h) - 2f(x_o)}{h^2} = f''(x_0).$$

(4) [10] **Outline** the proof of the Mean Value Theorem, filling in *all* the background, starting just after the Fundamental Lemma of Differentiation.

(5) [30] Prove that if $f(x)$ is continuous and strictly decreasing on an interval I, then $J := f(I)$ is also an interval, on which can be defined a function $g(y)$ for $y \in J$, that is, $g : J \to I$, such that $g(y)$ is strictly increasing, continuous, and inverse to $f(x)$.

(6) [10] Prove that the inverse function of x^3 exists for all $y \in \mathbb{R}$, and that it is differentiable for all $y \neq 0$, and NOT differentiable at $y = o$.

Quiz 3: July 11.

(1) [15] How did we use LUB to prove "Axiom C?" This is a hard question because it asks you to state the MAIN points in the proof.

(2) [15] State the Nested Intervals Theorem and use it to prove that if $\{[a_n, b_n]\}$ is a sequence of intervals that satisfies the conditions of the Nested Intervals Theorem, then, if $\{x_n\}$ is a sequence such that for all n, $x_n \in [a_n, b_n]$, then $\lim_{n\to\infty} x_n$ exists. What is the value of the limit? Justify your answer!

(3) [15] State the Bolzano-Weierstrass Theorem. Use the *idea* of its proof to prove that if x is a real number and $0 < x < 1$ then there exists a sequence $\{r_n\}$ of rational numbers such that $r_n \to x$ as $n \to \infty$.

(4) [15] State and prove the Boundedness Theorem (using Bolzano-Weierstrass).

(5) [15] How does the definition of "f is uniformly continuous on $[a, b]$" differ "in logic," from the definition ("in logic,") of "f is continuous on $[a, b]$?"

(6) [15] Define *Cauchy sequence* and prove that a convergent sequence is Cauchy.

Quiz 2: June 26.

(1) [10] Write out the definition of "$f(x)$ has a limit at $x = a,''$ in "logic" format, showing all needed quantifiers in their proper places.

(2) [5] Find $\lim_{x\to 2} \frac{x^3 - 4x^2 + 3x + 2}{x - 2}$. Hint: use long division.

(3) [20] Prove that your answer to #2 is correct.

(4) [15] Prove that $x^2 = 3$ has no rational solutions.

Note: In the next two questions, there are no formulas to work with so you have to work with Definitions. But you can also work with Theorems!

(5) [15] Suppose that

$$\lim_{x\to a} g(x) = 0;$$
$$g(x) \neq 0 \text{ if } x \neq a;$$
$$R := \lim_{x\to a} \frac{f(x)}{g(x)} \text{ exists.}$$

Prove that $\lim_{x\to a} f(x) = 0$.

(6) [15] In #5, suppose in addition that $R \neq 0$. Prove that $\lim_{x\to a} \dfrac{g(x)}{f(x)}$ exists. What is the value of the limit?

Quiz 1: June 19.

(1) [10] Prove that $1/(1/a) = a$ for all real $a \neq 0$.

(2) [15] Prove that $1 \in P$.

(3) [20] Prove that if $ab = 0$ then $a = 0$ or $b = 0$.

(4) [10] Prove that $(A \Rightarrow B) \iff ((\sim B) \Rightarrow (\sim A))$ is a tautology.

Note: $(\sim B) \Rightarrow (\sim A)$ is called the *contrapositive* of $A \Rightarrow B$.

(5) [5] State the contrapositive of #3. Is there any need to prove it, assuming #3 has been proved?

(6) [10] Let $f(x)$ be defined for $x \neq 1$ by $f(x) := \dfrac{x^2-1}{x-1}$. Find L so that, if we let $f(1) := L$, then $f(x)$ is continuous at $x = 1$.

(7) [20] Prove Theorem 2.6.