REGULARIZING EFFECTS OF NONLINEAR DAMPING IN NONLINEAR WAVE EQUATIONS

Abstract. The results of Strauss, Rauch, Struwe, Grillakis, Shatah-Striwe yield global well-posedness and regularity for the equation \( u_{tt} - \Delta u + u^5 = 0 \) in \( \mathbb{R}^3 \) (the so called critical case). The existence of smooth solutions in the supercritical case \( p > 1 + \frac{4}{n-2} \) appears to be an open problem, even for the space dimension \( n = 3 \). We show that semi-linear wave equations with a conveniently chosen nonlinear damping term \( g(u_t) \) and with defocusing smooth nonlinearities \( |u|^{p-1}u \) with supercritical growth \( p > 5 \) are globally well-posed in radially symmetric Sobolev spaces \( H^k_{rad}(\mathbb{R}^3) \times H^{k-1}_{rad}(\mathbb{R}^3) \) for all integer \( k \geq 3 \). We emphasize that the damping is not stronger than the nonlinearity and does not depend on the supercritical growth of the nonlinearity. The tools required for the proof include nonconcentration arguments, the \( L^1_t L^2_x - L^2_t L^\infty_x \) Strichartz estimate (valid for radial data) and remarkable cancellations of some potentially singular terms in the space-time estimates due to the presence of the damping term. Finally, we obtain scattering results for radial initial data in Sobolev spaces with \( k \geq 3 \).