MATH 3283 W HOMEWORK #6 DUE 4/20/10

CHAPTER 4, SECTIONS 4.4 – 4.6

PART A

1. DETERMINE IF $\sum_{n=1}^{\infty} \sin \frac{1}{n^2}$ CONVERGES OR DIVERGES.

GIVE REASONS FOR YOUR ANSWER

2. (a) SHOW THAT $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \cdot 2^n}$ CONVERGES.

GIVE REASONS FOR YOUR ANSWER.

(b) IF $\sum_{n=1}^{\infty} \frac{(-1)^{n+1}}{n \cdot 2^n} = L$, FIND THE SMALLEST

INTEGER n_0 SUCH THAT $|S_n - L| < \frac{1}{10^3}$

3 DETERMINE IF THE FOLLOWING SERIES IS ABSOLUTELY CONVERGENT, CONDITIONALLY CONVERGENT OR DIVERGENT. GIVE REASONS FOR YOUR ANSWERS.

$$1 - \frac{1 \cdot 3}{3!} + \frac{1 \cdot 3 \cdot 5}{5!} - \frac{1 \cdot 3 \cdot 5 \cdot 7}{7!} + \cdots + (-1)^n \frac{1 \cdot 3 \cdot 5 \cdot \ldots \cdot (2n+1)}{(2n+1)!} + \cdots$$

4 SUPPOSE $\sum_{n=1}^{\infty} a_n$ SATISFIES $a_1 = 1$, $a_{n+1} = \left(\sum_{j=1}^{n} 3^{-j}\right) a_n$

DOES $\sum_{n=1}^{\infty} a_n$ CONVERGE OR DIVERGE? GIVE

REASONS FOR YOUR ANSWER

PART B.

5 LET $\sum a_n$, $\sum b_n$ BE SERIES OF POSITIVE TERMS

(a) SUPPOSE $\lim\limits_{n\to\infty} \frac{a_n}{b_n} = 0$, PROVE THAT IF $\sum b_n$ IS CONVERGENT, THEN $\sum a_n$ IS CONVERGENT

(b) STATE THE CONVERSE OF (a). PROVE THAT IT IS TRUE, OR GIVE AN EXAMPLE TO SHOW THAT IT IS FALSE.

6 DETERMINE IF THE FOLLOWING SERIES CONVERGE OR DIVERGE, GIVE CAREFUL REASONS

(a) $\displaystyle\sum_{n=2}^{\infty} \frac{1}{n^{1.01} - n^{0.99}}$

(b) $\displaystyle\sum_{n=0}^{\infty} a_n$, WITH $a_0 = 1$

$a_{n+1} = \frac{(LN(n))^2}{n} a_n$

$LN(x) = \log_e(x)$

(c) $\displaystyle\sum_{n=1}^{\infty} \frac{n^n}{(2n)!}$

7 DETERMINE IF THE FOLLOWING SERIES ARE ABSOLUTELY CONVERGENT, CONDITIONALLY CONVERGENT OR DIVERGENT GIVE CAREFUL AND COMPLETE REASONS

(a) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \left(\sin \frac{1}{n}\right)^2$

(b) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n^2} \cdot n^{-4/5}$

(c) $\displaystyle\sum_{n=1}^{\infty} (-1)^{n+1} \frac{n}{n^2+1}$