1. Prove the following results
   (a) Let $k \in \mathbb{Z}^+$ and let $A \subseteq \mathbb{Z}^+$ such that
      1. $k$ is the smallest element of $A$
      2. If $n \in A$, then $n+1 \in A$

      Then $A = \{n \in \mathbb{Z}^+ | \ n \geq k\}$.

   (b) (Theorem 4.13 on page 22). Let $k \in \mathbb{Z}^+$ and let be a statement $P(n)$ depending on $n \in \mathbb{Z}^+$. Assume that
      1. $P(k)$ is true,
      2. $\forall n \in \mathbb{Z}^+$, if $P(n)$ is true, then $P(n+1)$ is true.

      Then $\forall n \geq k$, $P(n)$ is true.

2. (a) Use induction to prove that the following formula holds for any $n \in \mathbb{N}$:

$$1^3 + 2^3 + \cdots + n^3 = \left(\frac{n(n+1)}{2}\right)^2.$$

   (b) Consider the following implicit definition for a function $f : \mathbb{N} \to \mathbb{N}$,

$$\frac{1}{1 \cdot 2} + \frac{1}{2 \cdot 3} + \cdots + \frac{1}{n(n+1)} = 1 - f(n).$$

   Find a closed formula for $f(n)$ (i.e, an expression for $f(n)$ in which the number of terms does not vary with $n$), and prove your result using induction.

   (c) Find a closed formula for the sum

$$2^3 + 4^3 + 6^3 + \cdots + (2n)^3;$$

   be sure to prove that your formula works (using induction or other means).