1. Let $\{a_n\}$, and $\{b_n\}$ be sequences. Determine if the following are true or false. If true, give a proof. If false, give an example to show that it is false.
   (a) If $\{a_n\}$ converges and $\{a_nb_n\}$ converges, than $\{b_n\}$ converges.
   (b) If for every n, $a_n > 0$ and $\lim_{n\to\infty} a_n = L$, with $L > 0$, and $\{a_nb_n\}$ converges, then $\{b_n\}$ converges.
   (c) If $\{a_n\}$ converges to 0 and $\{b_n\}$ is bounded, then $\{a_nb_n\}$ converges.
   (d) If for every n, $a_n > 0$ and $\lim_{n\to\infty} a_n = L$, with $L < 1$, then there exists an $n_0 \in \mathbb{N}$ such that $n \geq n_0 \Rightarrow a_n < 1$.
   (e) If $\{a_n\}$, and $\{b_n\}$ converge, then $\{\cos(a_n^2 b_n^3)\}$ converges.

2. (a) Prove the Decreasing Monotonic Convergence Theorem: If $\{a_n\}$ is a decreasing sequence bounded below, then $\{a_n\}$ converges.
   (b) Let $a_n = e^{\frac{n+3}{n+1}}$.
      i Show that $\{a_n\}$ is decreasing and bounded below.
      ii Find the limit of $\{a_n\}$ as $n \to \infty$.