MATH 3283 W PROFESSIONAL PROBLEM #4
CHAPTER 4, POWER SERIES, 4.7 — 4.9

1. FIND THE RADIUS AND INTERVAL OF CONVERGENCE OF $\sum_{n=1}^{\infty} \frac{1}{n^2 b^{2n}} (x-a)^n$, WHERE a, b REAL, $b \neq 0$. SHOW ALL YOUR REASONING.

2. a) DETERMINE THE FIRST 7 TERMS OF THE POWER SERIES AT $x=0$ OF $f(x) = e^{x^2+x^3}$

HINT WE KNOW THAT $e^{a+b} = e^a \cdot e^b$.

b) WHAT IS THE RADIUS OF CONVERGENEE OF THE POWER SERIES AT $x=0$ FOR $g(x) = (2x+3x^2)e^{x^2+x^3}$? GIVE REASONS FOR YOUR ANSWER.

c) FIND THE POWER SERIES AT $x=0$ FOR $f(x) = 3x^{17} - 9x^{11} + x^7 - 5x^4 + x - 3$. GIVE REASONS FOR YOUR ANSWER.

3. a) SHOW THAT FOR ALL REAL x, $|x| = (x^2)^{1/2}$

b) FIND A POWER SERIES FOR $f(x) = \cos\sqrt{|x|}$ AT $x=0$. WHERE DOES IT CONVERGE? GIVE CAREFUL REASONS FOR YOUR ANSWERS

4. SUPPOSE $\sum a_n x^n$ IS A POWER SERIES FOR WHICH THE FOLLOWING FACTS ARE TRUE:

- $\sum a_n x^n$ CONVERGES ABSOLUTELY ON $(-1, 1)$
- $\sum a_n x^n$ CONVERGES AT $x = -\frac{3}{2}$ AND DIVERGES AT $x = 3$

A. DETERMINE IF THE FOLLOWING ARE TRUE, FALSE, OR THERE IS INSUFFICIENT INFORMATION GIVE CAREFUL AND COMPLETE REASONS

1) $\sum a_n$ CONVERGES

2) $\sum a_n x^n$ CONVERGES AT $x = \frac{5}{4}$

3) $\sum a_n x^n$ DIVERGES AT $x = -3$

B. GIVE UPPER AND LOWER BOUNDS FOR THE RADIUS OF CONVERGENCE OF $\sum a_n x^n$ GIVE CAREFUL REASONS

C. SUPPOSE IN ADDITION, IT IS TRUE THAL

- $|a_n(-2)^n| \leq \left(\frac{4}{5}\right)^n$ IF $n \geq 100$

WOULD THIS CHANGE YOUR ANSWER TO B? WHY? SHOW YOUR REASONING