Math 1155, Fall 2009, Exam I Solutions

Name: **Section:**

Instructions: This is the first exam for Math 1155, Intensive Precalculus. You have 50 minutes to complete the test. Do not start until you are told to begin.

When you receive this booklet, count the pages to be sure that you have every page. There should be 8 pages, including this cover sheet. No notes or books are allowed on this exam. Scientific calculators are allowed, however, calculators with graphing capabilities may not be used. You should simplify all fractions and square roots when they appear in your answer. For decimal answers, round angles to at least 1 decimal place and other numbers to 3 significant figures.

I expect you to use notation correctly and may penalize you for failing to do so. In particular, an equal sign should appear between two things that are equal; an equal sign should not appear between two things that are not equal. For full credit on a problem you must show the final correct answer and give a reasonably neat and logical account of how you got that answer.

There are a total of 50 points, distributed among 9 problems. The problems are worth varying amounts. You must show your work for all problems. Little or no credit will be given for unsupported answers. Even if you can do the problems in your head, you must convince me that you know what you're doing. Good luck.

Problem	Points	Possible
1-5		20
6		5
7		5
8		8
9		12
Total		50

This is the multiple choice portion of the exam. Circle all answers that are correct. There may be more than one correct answer to a question; if this is the case, all correct answers must be circled for credit. No partial credit on these.

1. (4 points) Which angles have a sine of $\frac{1}{2}$?

(a) $30°$

(b) $60°$

(c) $120°$

(d) $150°$

(e) None of these

Solution: $\arcsin \frac{1}{2} = 30°$, so the angle is either $30°$ or $180° - 30° = 150°$. Therefore, (a) and (d) is the correct answer.

2. (4 points) Which angles have a cosine of $\frac{1}{2}$?

(a) $30°$

(b) $60°$

(c) $120°$

(d) $150°$

(e) None of these

Solution: $\arccos \frac{1}{2} = 60°$. Therefore (b) is the correct answer.

3. (4 points) Assume that $\phi + \theta = 90°$. Which of the following statements are true? (Hint: draw a right triangle with angle θ.)

(a) $\sin\theta = \cos\phi$

(b) $(\sin\theta)^2 + (\cos\theta)^2 = 1$

(c) $\sin\phi = \sin\theta$

(d) $\cos\phi = -\cos\theta$

(e) None of these

Solution: Drawing a right triangle gives us angles of θ and ϕ. (a) is true since the opposite side to θ is the adjacent side to ϕ. (b) is true; I proved this in class. (c) and (d) are false... see what happens to them when θ is $30°$. (a) and (b) is the correct answer.

4. (4 points) How many triangles result from the information $a = 8$, $c = 6$ and $C = 35°$?

(a) 0

(b) 1

(c) 2

(d) None of these

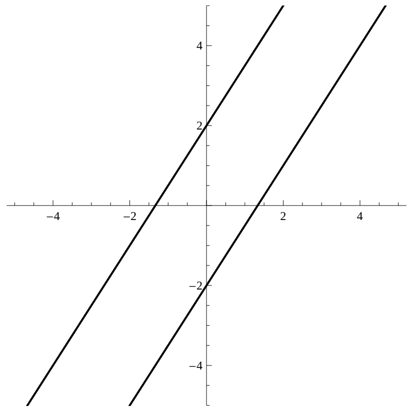
Solution: Use the law of sines to find A:

$$\frac{\sin 35°}{6} = \frac{\sin A}{8}$$

$$\sin A \approx .7648.$$

Thus, $A = 49.89°$ or $A = 130.11°$. These both give valid triangles since the sum of the angles didn't exceed $180°$.

5. (4 points) What symmetries does the following graph exhibit?



(a) x-axis

(b) y-axis

(c) origin

(d) None of these

Solution: This just has origin symmetry. The answer is (c).

6. (5 points) The shadow cast by a flagpole is 5 meters long when the sun is 25° above the horizon. How tall is the flagpole?

Solution: Draw a right triangle with a 25° angle whose adjacent (horizontal) side is 5 meters. We want the opposite (vertical) side (call it x), so we use tangent:

$$\tan 25^\circ = \frac{x}{5}.$$

Solving, we find that $x = 2.33$ meters.

7. (5 points) Solve any triangles that may result from $A = 30^\circ$, $b = 4$, $c = 10$.

Solution: First, use the law of cosines to find a:

$$a^2 = b^2 + c^2 - 2bc\cos A$$

$$a^2 = 16 + 100 - 80\frac{\sqrt{3}}{2}$$

$$a \approx 6.8351.$$

Use the law of cosines again to find C:

$$c^2 = a^2 + b^2 - 2ab\cos C$$

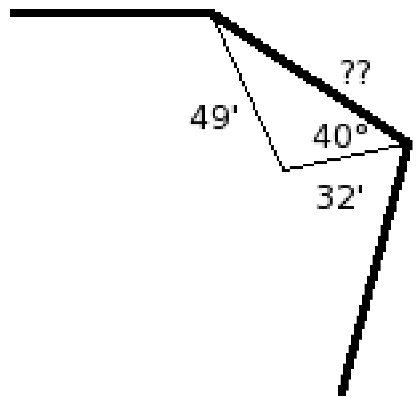
$$\cos C = \frac{a^2 + b^2 - c^2}{2ab} \approx -.6817$$

$$\cos C \approx 133.0^\circ.$$

Finally, B can be found by the sum of the angles:

$$B = 180^\circ - 30^\circ - 133^\circ = 17^\circ.$$

8. (8 points) A surveyor is measuring the boundary of a property. Because of various obstacles like hedges and a statue, the surveyor can only manage to collect the information below. Can you determine the length of the side of property in question? If so, do it; if not, find any possible lengths of the side.



Solution: Let $A = 40°$, $a = 49'$ and $b = 32'$. Then find B via the law of sines:

$$\frac{\sin 40°}{49} = \frac{\sin B}{32}$$

We find that $B = 24.8°$ or $B = 155.2°$. In fact $B = 155.2°$ is too big of an angle since it makes the sum of the angles bigger than $180°$. So, $B = 24.8°$.

Using the sum of the angles, we also see that

$$C = 180° - 40° - 24.8° = 115.2°.$$

Finally, use the law of sines to find c, which is the desired side:

$$\frac{\sin 115.2°}{c} = \frac{\sin 40°}{49}$$

$$c = \frac{49 \sin 115.2°}{\sin 40°} = 69.0'.$$

9. (12 points) This problem will determine the exact value of $\sin 75°$. Decimal answers will not get full credit.

(a) Draw a $45° - 60° - 75°$ triangle with a side of length 1 opposite the $60°$ angle. Find the exact length of the side opposite the $45°$ angle.

(b) Draw a new picture of the triangle, and split the $75°$ angle by drawing a line perpendicular to opposite side through the vertex of the angle. Find the exact length of this new line segment. (Hint: you now have two familiar triangles side by side.)

(c) Using your picture from part (b), find the exact lengths of the two parts of the side opposite the $75°$ angle. Add them up.

(d) You are now in a position to solve for $\sin 75°$ exactly. Do so. (If you're confused here, look at your picture from part (a).)

Solution: (a) Use the law of sines to find the indicated side:

$$\frac{\sin 60°}{1} = \frac{\sin 45°}{x}.$$

Thus,

$$x = \frac{\sin 45°}{\sin 60°} = \frac{\frac{\sqrt{2}}{2}}{\frac{\sqrt{3}}{2}} = \sqrt{\frac{2}{3}}.$$

(b) The new line creates a $45° - 45° - 90°$ triangle with hypotenuse 1. The length must therefore be $\frac{1}{\sqrt{2}}$.

(c) The other leg of the $45° - 45° - 90°$ triangle has length $\frac{1}{\sqrt{2}}$. The other part is the leg of the $30° - 60° - 90°$ triangle opposite the $30°$ angle, and we get its length by dividing the answer to (b) by $\sqrt{3}$, which gives us $\frac{1}{\sqrt{6}}$. Thus, the total length is

$$\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}.$$

(d) Use the law of sines:

$$\frac{\sin 75°}{\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}} = \frac{\sin 60°}{1}.$$

Thus,

$$\sin 75° = \frac{\sqrt{3}}{2}\left(\frac{1}{\sqrt{2}} + \frac{1}{\sqrt{6}}\right) = \frac{\sqrt{3}}{2\sqrt{2}} + \frac{1}{2\sqrt{2}} = \frac{\sqrt{3}+1}{2\sqrt{2}} = \frac{\sqrt{6}+\sqrt{2}}{4}.$$