Math 1155, Fall 2009, Exam II

Name: **Section:**

Instructions: This is the second exam for Math 1155, Intensive Precalculus. You have 50 minutes to complete the test. Do not start until you are told to begin.

When you receive this booklet, count the pages to be sure that you have every page. There should be 8 pages, including this cover sheet. No notes or books are allowed on this exam. Scientific calculators are allowed, however, calculators with graphing capabilities may not be used. You should simplify all fractions and square roots when they appear in your answer. For decimal answers, round angles to at least 1 decimal place and other numbers to 3 significant figures.

I expect you to use notation correctly and may penalize you for failing to do so. In particular, an equal sign should appear between two things that are equal; an equal sign should not appear between two things that are not equal. For full credit on a problem you must show the final correct answer and give a reasonably neat and logical account of how you got that answer.

There are a total of 50 points, distributed among 9 problems. The problems are worth varying amounts. You must show your work for all problems. Little or no credit will be given for unsupported answers. Even if you can do the problems in your head, you must convince me that you know what you're doing. Good luck.

Problem	Points	Possible
1-5		20
6		5
7		5
8		8
9		12
Total		50

This is the multiple choice portion of the exam. Circle all answers that are correct. There will only be one correct answer to a question. No partial credit on these.

1. (4 points) Which of the following equals $\cos(-\frac{\pi}{6})$?

(a) $\cos\frac{5\pi}{6}$

(b) $\cos\frac{7\pi}{6}$

(c) $\sin\frac{\pi}{3}$

(d) $\cos\frac{\pi}{3}$

(e) None of these

Solution: $\cos(-\frac{\pi}{6}) = \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$. The answer is thus (c).

2. (4 points) What is the graph of the equation $2x = x^2 + y^2$?

(a) A circle of radius 1 centered at (1,0)

(b) A circle of radius 1 centered at (0,1)

(c) A circle of radius 2 centered at (0,1)

(d) A parabola with vertex at (1,1)

(e) None of these

Solution: Move everything to one side:

$$0 = x^2 - 2x + y^2.$$

Now, complete the square:

$$0 = (x^2 - 2x + 1) + y^2 - 1$$
$$0 = (x - 1)^2 + y^2 - 1.$$

Move the constant to the other side:

$$1 = (x - 1)^2 + y^2.$$

It's a circle of radius 1 centered at (1,0). The answer is (a).

3. (4 points) How many degrees is $\frac{7\pi}{10}$ radians?

(a) 56

(b) 98

(c) 126

(d) 154

(e) None of these

Solution: Multiply $\frac{7\pi}{10}$ by $\frac{180}{\pi}$. Result: 126 degrees. The answer is (c).

4. (4 points) If $\sin\theta = \frac{3}{4}$ and θ is in the second quadrant, then what is $\tan\theta$?

(a) $\frac{3}{\sqrt{7}}$

(b) $-\frac{3}{\sqrt{7}}$

(c) $\frac{\sqrt{7}}{3}$

(d) $-\frac{\sqrt{7}}{3}$

(e) None of these

Solution: First, notice that we're in the second quadrant. Therefore, $\cos\theta < 0$. Now, find $\cos\theta$:

$$\begin{aligned}\sin^2\theta + \cos^2\theta &= 1\\ \frac{9}{16} + \cos^2\theta &= 1\\ \cos^2\theta &= \frac{7}{16}\\ \cos\theta &= -\frac{\sqrt{7}}{4}.\end{aligned}$$

To complete the problem:

$$\tan\theta = \frac{\sin\theta}{\cos\theta} = \frac{\frac{3}{4}}{-\frac{\sqrt{7}}{4}} = -\frac{3}{\sqrt{7}}.$$

The answer is (b).

5. (4 points) Let $f(x) = -2\cos(\frac{4\pi}{3}x)$. What are the amplitude and period?

(a) Amplitude: -2, Period: 4/3

(b) Amplitude: 2, Period: 4/3

(c) Amplitude: 2, Period: 3/2

(d) Amplitude: -2, Period: 3/2

(e) None of these

Solution: The amplitude is 2. We get the period T thusly:

$$T = \frac{2\pi}{\frac{4\pi}{3}} = \frac{6}{4} = \frac{3}{2}.$$

The answer is (c).

6. (5 points) Fill in this table with exact values.

	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$
sin	0	$\frac{1}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{\sqrt{3}}{2}$	1
cos	1	$\frac{\sqrt{3}}{2}$	$\frac{\sqrt{2}}{2}$	$\frac{1}{2}$	0

7. (5 points) A bicycle tire is 50cm in radius, and the bicycle that it is attached to is traveling 10 m/s. Find how fast the tire is rotating in radians per second.

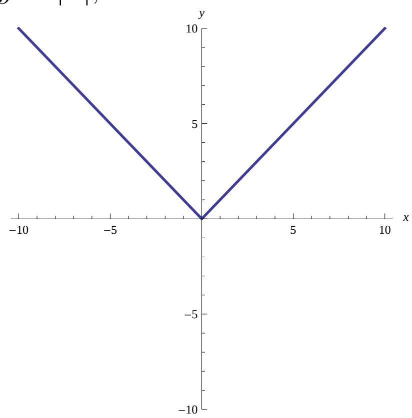
Solution: This is a $v = r\omega$ problem with $v = 10$m/s and $r = 50$cm. Let's convert r to meters so that we have consistent units: $r = .5$m. Now,

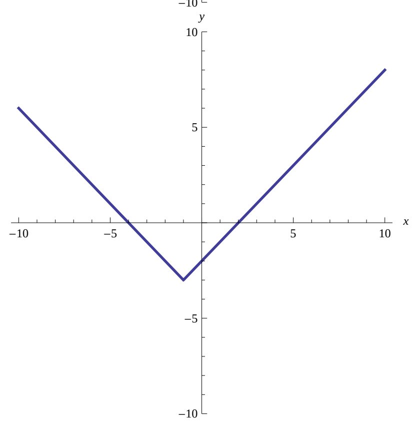
$$\omega = \frac{v}{r} = \frac{10}{.5} = 20.$$

Thus, $\omega = 20$ radians per second.

8. (8 points) Graph $y + 3 = |-(x+1)|$ by first graphing $y = |x|$ and then applying transformations. Label clearly what equation is in each graph.

Solution: Let's do some algebra first: $|-(x+1)| = |x+1|$ since the absolute value function is even. Now we're trying to plot $y + 3 = |x+1|$, so the first graph below shows the graph of $y = |x|$, and we translate this 3 units down and one unit left to get the answer.





9. (12 points) Consider the functions

$$f(x) = \frac{1}{\sqrt{x+3}}$$

$$g(x) = \frac{1}{x^2} - 3.$$

(a) Find the domain and range of $f(x)$. (Hint: For the range, first ask yourself "What are the possible outputs of $\sqrt{x}$.")

(b) Find $f^{-1}(x)$ and its domain.

(c) Find the domain and range of $g(x)$. (See previous hint, and adapt it)

(d) Are $f(x)$ and $g(x)$ inverses? Explain.

Solution: (a) The domain of $f(x)$ is $-3 < x$ (to avoid square roots of negative numbers and dividing by zero).

The possible outputs of $\sqrt{x+3}$ are all positive numbers, and so the possible outputs of $f(x) = \frac{1}{\sqrt{x+3}}$ are also all positive numbers. Therefore, the range of $f(x)$ is $0 < y$.

(b) Start with

$$y = \frac{1}{\sqrt{x+3}}.$$

Swap the x's and y's:

$$x = \frac{1}{\sqrt{y+3}}.$$

Solve this for y:

$$\frac{1}{x} = \sqrt{y+3}$$

$$\frac{1}{x^2} = y+3$$

$$\frac{1}{x^2} - 3 = y.$$

Thus,

$$\frac{1}{x^2} - 3 = f^{-1}(x).$$

The domain of $f^{-1}(x)$ must match the range of $f(x)$, and so the domain is $0 < x$.

(c) The domain of $g(x)$ is $x \neq 0$, in order to avoid dividing by zero.

The possible outputs of x^2 are all positive numbers, and so the possible outputs of $\frac{1}{x^2}$ are also all positive numbers. Thus, the possible outputs of $g(x) = \frac{1}{x^2} - 3$ are all numbers bigger than -3. Therefore, the range of $g(x)$ is $-3 < y$.

(d) They are not inverses. If they were inverses, we would have $f(g(-1)) = -1$. However,

$$f(g(-1)) = f(-2) = 1.$$