Math 1155, Fall 2009, Exam III Solutions

Name: **Section:**

Instructions: This is the third exam for Math 1155, Intensive Precalculus. You have 50 minutes to complete the test. Do not start until you are told to begin.

When you receive this booklet, count the pages to be sure that you have every page. There should be 9 pages, including this cover sheet. No notes or books are allowed on this exam. Scientific calculators are allowed, however, calculators with graphing capabilities may not be used. You should simplify all fractions and square roots when they appear in your answer. For decimal answers, round angles to at least 1 decimal place and other numbers to 3 significant figures.

I expect you to use notation correctly and may penalize you for failing to do so. In particular, an equal sign should appear between two things that are equal; an equal sign should not appear between two things that are not equal. For full credit on a problem you must show the final correct answer and give a reasonably neat and logical account of how you got that answer.

There are a total of 50 points, distributed among 10 problems. The problems are worth varying amounts. You must show your work for all problems. Little or no credit will be given for unsupported answers. Even if you can do the problems in your head, you must convince me that you know what you're doing. Good luck.

Problem	Points	Possible
1-5		20
6		6
7		6
8		6
9		6
10		6
Total		50

This is the multiple choice portion of the exam. Circle all answers that are correct. There will only be one correct answer to a question. No partial credit on these.

1. (4 points) Find $(\frac{\sqrt{2}}{2} + \frac{\sqrt{2}}{2}i)^2$.

(a) $-i$

(b) $-1 - i$

(c) i

(d) $1 + i$

(e) None of these

Solution: Expand it out, simplify, and remember that $i^2 = -1$ to get i. The answer is (c).

2. (4 points) Find $\arcsin(\sin \frac{4\pi}{3})$.

(a) $\frac{\pi}{3}$

(b) $\frac{2\pi}{3}$

(c) $\frac{-\pi}{3}$

(d) $\frac{4\pi}{3}$

(e) None of these

Solution: $\sin \frac{4\pi}{3} = \sin(-\frac{\pi}{3})$ (look at a circle). So,

$$\arcsin(\sin \frac{4\pi}{3}) = \arcsin(\sin(-\frac{\pi}{3})) = -\frac{\pi}{3}.$$

The answer is (c).

3. (4 points) Find $\arccos(\cos\frac{4\pi}{3})$.

(a) $\frac{\pi}{3}$

(b) $\frac{2\pi}{3}$

(c) $\frac{-\pi}{3}$

(d) $\frac{4\pi}{3}$

(e) None of these

Solution:

$$\arccos(\cos\frac{4\pi}{3}) = \arccos(\cos\frac{2\pi}{3}) = \frac{2\pi}{3}.$$

The answer is (b).

4. (4 points) Where is the vertex of the function $f(x) = 2x^2 - 16x + 3$?

(a) (4,-13)

(b) (4,-29)

(c) (0,3)

(d) (2,-21)

(e) None of these

Solution: Complete the square:

$$f(x) = 2(x^2 - 8x) + 3$$

$$f(x) = 2(x^2 - 8x + 16 - 16) + 3$$
$$f(x) = 2((x-4)^2 - 16) + 3$$
$$f(x) = 2(x-4)^2 - 32 + 3$$
$$f(x) = 2(x-4)^2 - 29$$

The vertex is at (4,-29), so the answer is (b).

5. (4 points) Solve the equation $4\arcsin(2x) = \pi$.

(a) $x = 0$

(b) $x = \frac{\sqrt{2}}{4}$

(c) $x = \frac{1}{4}$

(d) $x = \frac{\sqrt{3}}{4}$

(e) None of these

Solution:

$$4\arcsin(2x) = \pi$$
$$\arcsin(2x) = \frac{\pi}{4}$$
$$2x = \sin(\frac{\pi}{4})$$
$$2x = \frac{\sqrt{2}}{2}$$
$$x = \frac{\sqrt{2}}{4}$$

The answer is (b).

6. (6 points) It is well known that there is a linear relationship between degrees Fahrenheit and degrees Celsius. Let $x = ^\circ C$ and $y = ^\circ F$.

a) Given $0^\circ C = 32^\circ F$ and $100^\circ C = 212^\circ F$, find the equation to convert $^\circ C$ to $^\circ F$.

Solution: Use slope intercept form. The intercept is 32, and the slope is $\frac{212-32}{100-0} = \frac{9}{5}$. The equation is thus

$$y = \frac{9}{5}x + 32$$

b) Find the number z for which $z^\circ C = z^\circ F$.

Solution: We want $y = x$ in the above equation.

$$x = \frac{9}{5}x + 32$$

$$-\frac{4}{5}x = 32$$

$$x = -40$$

7. (6 points) A parabolic arch is 90 feet high at the center and 80 feet high 10 feet from the center in either direction.

a) Find the equation of the arch (Hint: Let $x = 0$ at the center of the arch, and let $y = 0$ represent the ground).

Solution: We know three points on the parabola: (0,90), (10,80), (-10,80). We also know that (0,90) is the vertex.

Thus,

$$f(x) = a(x-0)^2 + 90$$

$$f(x) = ax^2 + 90$$

We need to find a, and we'll use the point (10,80) to do so:

$$80 = a10^2 + 90$$

$$-10 = a100$$

$$-\frac{1}{10} = a$$

So,

$$f(x) = -\frac{x^2}{10} + 90$$

b) How wide is the arch?

Solution: We'll need to find when the arch hits the ground. That is: when does $f(x) = 0$?

$$0 = -\frac{x^2}{10} + 90$$

$$-90 = -\frac{x^2}{10}$$

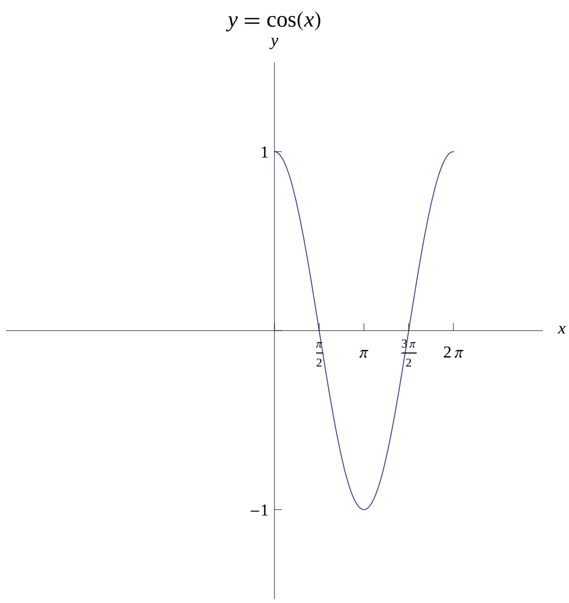
$$900 = x^2$$

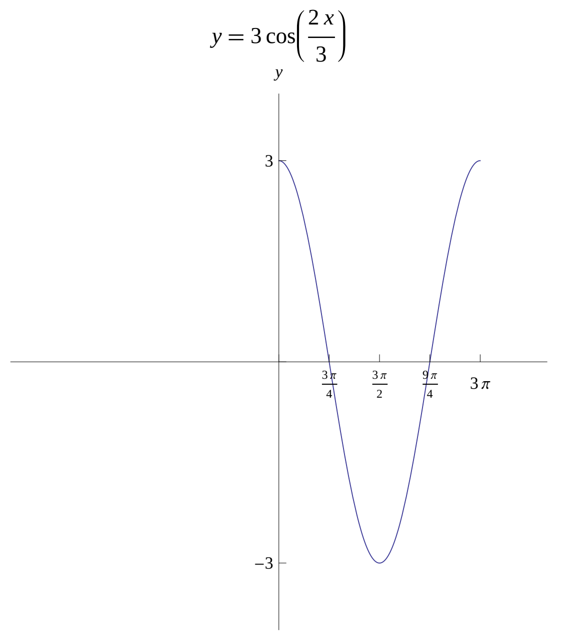
$$\pm 30 = x$$

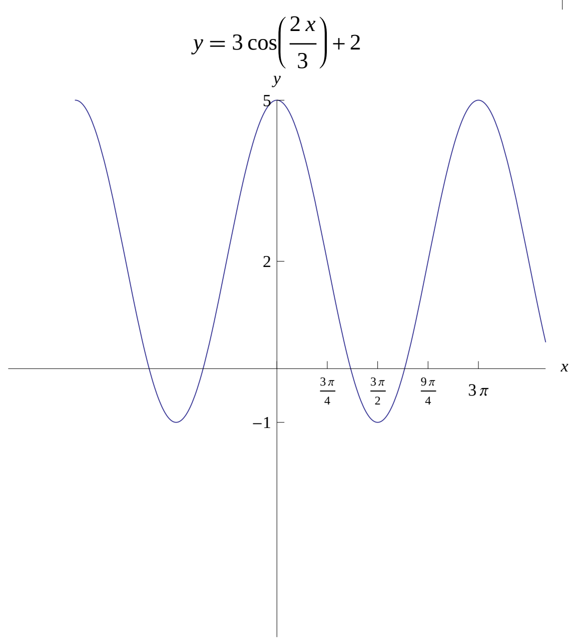
The distance between these two zeros is 60 feet.

8. (6 points) Graph $f(x) = 3\cos(\frac{2x}{3}) + 2$. Find the amplitude and period of this function. If you use multiple graphs, be sure to label each graph. You will also be required to clearly identify the usual 5 points on the final graph.

Solution: First of all, the amplitude is 3 and the period is 3π.







9. (6 points) A farmer has 100 feet of fence to enclose a rectangule. To increase the maximum area, the farmer will only fence three sides of the rectangle and use his barn as the fourth side. Let x be the length of the side perpendicular to the barn.

a) Find the area enclosed as a function of x.

Solution: The length of fence is $2x + y$, with x as above, and y denoting the length of the side parallel to the barn. Thus,

$$100 = 2x + y$$

$$100 - 2x = y$$

The area is given by

$$A(x) = xy = x(100 - 2x)$$

b) Find the maximum possible area that can be enclosed.

Solution: Complete the square on $A(x)$

$$A(x) = -2x^2 + 100x$$

$$A(x) = -2(x^2 - 50x)$$

$$A(x) = -2(x^2 - 50x + 625 - 625)$$

$$A(x) = -2((x - 25)^2 - 625)$$

$$A(x) = -2(x - 25)^2 + 1250$$

The maximum value of $A(x)$ is thus 1250 square feet.

10. (6 points) Let $f(x) = \cos(x+2) + 1$.

a) Find the inverse function $f^{-1}(x)$.

Solution: Start by swapping the x's and y's, then solve for y.

$$x = \cos(y+2) + 1$$

$$x - 1 = \cos(y+2)$$

$$\arccos(x-1) = y + 2$$

$$\arccos(x-1) - 2 = y$$

So, $f^{-1}(x) = \arccos(x-1) - 2$

b) Determine the domains of $f(x)$ and $f^{-1}(x)$ keeping in mind that $f(x)$ needs to be one-to-one.

Solution: We make $f(x)$ one-to-one by restricting its domain. Recall that we restrict the domain of $\cos(x)$ to be $0 \leq x \leq \pi$. On that basis, to get the domain of $f(x)$, we want

$$0 \leq x + 2 \leq \pi$$

$$-2 \leq x \leq \pi - 2$$

For $f^{-1}(x)$, remember that the domain of $\arccos(x)$ is $-1 \leq x \leq 1$. Similarly to above, we need

$$-1 \leq x - 1 \leq 1$$

$$0 \leq x \leq 2$$