Math 1155, Fall 2009, Exam IV Solutions

Name: **Section:**

Instructions: This is the fourth exam for Math 1155, Intensive Precalculus. You have 50 minutes to complete the test. Do not start until you are told to begin.

When you receive this booklet, count the pages to be sure that you have every page. There should be 8 pages, including this cover sheet. No notes or books are allowed on this exam. Scientific calculators are allowed, however, calculators with graphing capabilities may not be used. You should simplify all fractions and square roots when they appear in your answer. For decimal answers, round angles to at least 1 decimal place and other numbers to 3 significant figures.

I expect you to use notation correctly and may penalize you for failing to do so. In particular, an equal sign should appear between two things that are equal; an equal sign should not appear between two things that are not equal. For full credit on a problem you must show the final correct answer and give a reasonably neat and logical account of how you got that answer.

There are a total of 50 points, distributed among 9 problems. The problems are worth varying amounts. You must show your work for all problems. Little or no credit will be given for unsupported answers. Even if you can do the problems in your head, you must convince me that you know what you're doing. Good luck.

Problem	Points	Possible
1-5		20
6		8
7		10
8		6
9		6
Total		50

This is the multiple choice portion of the exam. Circle all answers that are correct. There will only be one correct answer to a question. No partial credit on these.

1. (4 points) Find the domain of

$$\ln \frac{x+1}{x-3}.$$

(a) $(-\infty, -1] \cup (3, \infty)$

(b) $(-1, 3)$

(c) $[-1, 3)$

(d) $(-\infty, -1) \cup (3, \infty)$

(e) None of these

Solution: Since we're taking a logarithm, we need $\frac{x+1}{x-3} > 0$.

The numerator of this is zero when $x = -1$ and the denomitator is zero when $x = 3$. Thus, there are three regions we're concerned with: $(-\infty, -1)$, $(-1, 3)$, and $(3, \infty)$. By looking at the factors in each region, we can see that $\frac{x+1}{x-3} > 0$ in the regions $(-\infty, -1)$ and $(3, \infty)$. Since the inequality is strict, we do not want any endpoints.

The answer is (d).

2. (4 points) Find the horizontal asymptote of $\frac{x^3-3x+71x-1}{5x^3+x-20}$.

(a) $x = \frac{1}{5}$

(b) $y = \frac{1}{5}$

(c) $x = 5$

(d) $y = 5$

(e) None of these

Solution: Let

$$f(x) = \frac{x^3 - 3x + 71x - 1}{5x^3 + x - 20}.$$

As x gets big, the numerator behaves like x^3 and the denominator behaves like $5x^3$. Thus, as $x \to \pm\infty$, $f(x) \to \frac{1}{5}$. The asymptote is $y = \frac{1}{5}$, so (b) is the correct answer.

3. (4 points) Find the vertical asymptote(s) of $\frac{x^2-5x+6}{(x+1)(x-10)(x-3)}$.

(a) $x = -1$, $x = 10$, $x = 3$

(b) $y = -1$, $y = 10$, $y = 3$

(c) $x = -1$, $x = 10$

(d) $y = 10$

(e) None of these

Solution: Let

$$f(x) = \frac{x^2 - 5x + 6}{(x+1)(x-10)(x-3)}.$$

Start by reducing to lowest terms:

$$f(x) = \frac{(x-3)(x-2)}{(x+1)(x-10)(x-3)}$$

we have a hole at $x = 3$ since we the factor $(x-3)$ appears in the numerator and denominator. The vertical asymptotes are thus $x = 10$ and $x = -1$, so the answer is (c).

4. (4 points) Simplify $\log_3 7 \log_7 27$.

(a) 3

(b) 9

(c) $\ln 9$

(d) $\ln(\frac{7}{3}) \ln(\frac{27}{7})$

(e) None of these

Solution: The only way out of this problem is the base change formula. The most elegent way is to convert $\log_7 27$ to base 3:

$$\begin{aligned}\log_3 7 \log_7 27 &= \log_3 7 \frac{\log_3 27}{\log_3 7} \\ &= \log_3 27 = 3\end{aligned}$$

The answer is (a).

5. (4 points) Find all of the roots of $3x^3 + 7x^2 + 8x + 2$. (Hint: Proceed by elimination)

(a) -3, -1+i, -1-i

(b) $-\frac{1}{3}$, -1+i, -1-2i

(c) $\frac{1}{3}$, -1+i, -1-i

(d) $-\frac{1}{3}$, -2, -1+i, -1-i

(e) $-\frac{1}{3}$, -1+i, -1-i

Solution: Proceeding by elimination, (d) is obviously wrong: There are 4 roots listed and a degree 3 polynomial. (b) is also wrong since the two complex roots are not conjugate. (a) doesn't work since -3 fails the rational roots test.

The options are now (c) and (e). Is $\frac{1}{3}$ a root, or is it $-\frac{1}{3}$. In fact, $\frac{1}{3}$ can't be a root since plugging in $x = \frac{1}{3}$ yields a positive number (each term is positive, and we're doing no subtraction). This leaves (e) as the answer.

6. (8 points) Polonium-210 is a radioactive substance, hence it decays over time. Assume that at time $t = 1$ day, we have 200 grams of polonium-210, and at time $t = 3$ days we have 132 grams. Let $A(t)$ be the amount of polonium-210 at time t.

a) Find $A(t)$. (If you can't think of anything better to do, let $A(t) = A_0 e^{\lambda t}$ and solve for λ and A_0.)

Solution: We know that $A(1) = 200$ and $A(3) = 132$. Therefore,

$$A_0 e^{3\lambda} = 132$$

$$A_0 e^{\lambda} = 200$$

Divide the top equation by the bottom equation:

$$e^{2\lambda} = .66$$

$$2\lambda = \ln .66 = -.4155$$

$$\lambda = -.2078$$

It remains to find A_0. We currently have $A(t) = A_0 e^{-.2078t}$. Let's plug in $t = 1$:

$$200 = A_0 e^{-.2078}$$

$$200 = .8124 A_0$$

$$246.18 = A_0$$

Thus,

$$A(t) = 246.18 e^{-.2078t}$$

b) Determine the half-life of polonium-210. (That is, how long until only half of our initial amount remains.)

Solution: We have A_0 grams of polonium at time $t = 0$. We want to see when we have $\frac{A_0}{2}$ grams:

$$\frac{A_0}{2} = A_0 e^{-.2078t}$$

$$\frac{1}{2} = e^{-.2078t}$$

$$\ln(\frac{1}{2}) = -.2078t$$

$$-.6931 = -.2078t$$

$$3.3363 = t$$

So, the half-life is about 3.34 days.

Nick's note: in actual fact, polonium-210 has a half-life of 5 days. The problem was intended to have this for an answer, but I messed up.

7. (10 points) You are looking to invest \$1000, and two banks have made nice offers. Bank A will give you 6% interest compounded annually and bank B will give you 5.5% interest compounded continuously. Which bank do you choose? (A lucky guess is worth nothing. We'll need to see a comparison of how much would be in each account after some time has elapsed.)

Solution: It suffices to look at the balances after 1 year. Let A be the balance in bank A after 1 year and B the balance in bank B.

For bank A, we have $n = 1$, $r = .06$, $t = 1$ and $P = \$1000$. Therefore

$$A = \$1000(1 + .06) = \$1060$$

For bank B, we have continuous interest with $r = .055$, $t = 1$ and $P = \$1000$. Therefore

$$B = \$1000e^{.055} = \$1056.54$$

Bank A is better.

8. (6 points) Solve the equation $\ln(x+4) + \ln(x+2) = \ln 8$

Solution: Start by combining the logarithms on the left side:

$$\ln((x+4)(x+2)) = \ln 8.$$

Now, take e to the power of both sides:

$$(x+4)(x+2) = 8$$

$$x^2 + 6x + 8 = 8$$
$$x^2 + 6x = 0$$
$$x(x+6) = 0.$$

Thus, $x = 0$ or $x = -6$. Plugging these in to check, we see that $x = 0$ is OK, while $x = -6$ is not (end up taking ln of a negative number).

The answer is thus $x = 0$.

9. (6 points) $2+i$ is a root of $x^5 - 3x^4 + 2x^3 + 2x^2 + x + 5$. Find all of the other roots.

Solution: Since $2+i$ is a root, $2-i$ must also be a root. By the rational roots test, 1, -1, 5, and -5 may be roots. Plugging these in reveals that $x = -1$ is a third root. Therefore,

$$(x-(2+i))(x-(2-i))(x+1)$$

is a factor of

$$x^5 - 3x^4 + 2x^3 + 2x^2 + x + 5.$$

Now we'll expand out the factor and do some long division.

$$\begin{aligned} &(x-(2+i))(x-(2-i))(x+1) \\ &= (x^2 - 4x + 5)(x+1) \\ &= x^3 - 3x^2 + x + 5 \end{aligned}$$

By long division, we get

$$x^5 - 3x^4 + 2x^3 + 2x^2 + x + 5 = (x^3 - 3x^2 + x + 5)(x^2 + 1).$$

In remains to find the roots of $x^2 + 1$. They are i and $-i$.
The complete set of roots is therefore $2+i$, $2-i$, i, $-i$, and -1.