

Syllabus
Stochastic processes, Math 8659, Fall 2009

Lectures: 10:10-11:00 MWF VinH 209
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Office hours: MWF, 13:25-14:15
Textbook: Lecture notes will be provided
Final examination: Take home final due on Tuesday, December 22, 2009

PREREQUISITE KNOWLEDGE: Basics of the Itô stochastic integration and continuous time martingales.

APPROXIMATE OUTLINE OF THE COURSE: The course will be about the relations between diffusion processes (solutions of Itô stochastic equations) and second-order linear elliptic and parabolic differential equations. The goal is to show how probabilistic methods help obtain important information about solutions of PDEs.

A few homeworks will be assigned and will form part of the final grade.

I will take the material for the course from my two publications:

“Probabilistic methods of investigating interior smoothness of harmonic functions associated with degenerate elliptic operators”, Centro di ricerca matematica Ennio De Giorgi, Scuola Normale Superiore, Pubblicazione della classe di scienze, Pisa, 2004;

On Kolmogorov’s equations for finite dimensional diffusions, pp. 1-63 in “CIME courses”, Lecture Notes in Mathematics, Vol. 1715, Springer-Verlag, 1999.

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