

Syllabus

Topics in PDE, Math 8590, Fall 2000

Lectures:	1115-1205 MWF VinH 113
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Office hours:	MWF, 1325-1415
Textbook:	Lecture notes will be provided
Prerequisites:	Some knowledge of functional analysis, Fourier transform, and integration theory
Final examination:	Take home final due on Dec 16, 2000

The course is intended to cover essentials of the L_p theory of elliptic and parabolic equations in the whole space as well as in smooth domains with Dirichlet and Neumann boundary conditions. The idea is to make the course self contained, in particular, the Calderón-Zygmund theorem will be proved. The contents of the lecture notes which are ready at this moment is included below (as of July 31). It is constantly changing. If time permits, divergent form second order equations will be included.

Five homeworks will be assigned and will form part of the final grade.

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