

Preliminary Syllabus Stochastic Processes

Lectures:	10:10am-11:00 am, VinH 311
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Office hours:	MWF, 13:25-14:15
Textbook:	Lecture note will be provided For the scope see the table of contents below
Prerequisites:	Math 8652
Final examination:	to be announced

Homeworks and Midterms: There will be one midterm and 7 homework assignments. The dates will be specified later.

Depending on time and students' interests and knowledge parts of the following will be covered.

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