

Topics in Probability, Math 8660, Fall 2002
 Controlled diffusion processes
 Syllabus

Lectures: 11:15-12:05 MWF VinH 211
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 Office hours: MWF, 14:00-15:00
 Textbook: Lecture notes will be provided
 Prerequisites: Itô stochastic integral, martingales
 Final examination: Take home final due ??-??

The course is intended to cover essentials of Controlled diffusion processes approximately outlined in the table of contents below.

Five homeworks will be assigned and will form part of the final grade.

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