

Syllabus
 Topics in PDE, Math 8590, Fall 2004
 Elliptic equations in L_p -spaces

Lectures: 2:30-3:20 MWF VinH 6
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 Office hours: MWF, 1325-1415
 Textbook: Lecture notes will be provided
 Prerequisites: Some knowledge of functional analysis,
 Fourier transform, and integration theory
 Final examination: Take home final due on Friday Dec 17, 2004

The course is intended to cover essentials of the L_p theory of elliptic equations in the whole space as well as in smooth domains with Dirichlet and Neumann boundary conditions. The idea is to make the course self contained, in particular, the Calderón-Zygmund theorem or/and the Stampacchia interpolation theorem will be proved.

Five homeworks will be assigned and will form part of the final grade.

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