

Preliminary Syllabus Prediction and Filtering

Lectures:	12:20-13:10 MWF, FordH B80
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Office hours:	MWF, 13:25-14:15
Textbook:	Lecture note will be provided For the scope see the table of contents below
Prerequisites:	Math 5651 or Stat 5101
Final examination:	to be announced

Homeworks and Midterms: There will be two midterms and 7 homework assignments. The dates will be specified later.

Homeworks should be written clearly and more or less neatly using one side of each sheet of paper. The total possible score on each assignment will be 36. No late assignment will be permitted. The best 6 assignments will be chosen for each student, giving a total possible score of 216. The raw score will be then divided by 6 to give a net score, the best possible net score being 36. You can miss one and only one assignment with only the modest penalty of then having the other 6 being the best six.

Evaluation: The grade for the semester will be based on 144 possible points, 72 of which will be on the final examination. The other 72 will come from two possible 36-points contributions, one being the net score for homeworks and another being a rescaled result of two 115-minute midterm examinations. The rescaling may be made in order to take into account a possible overestimating of the level of students, which could cause low grades. The rescaling will result in multiplying the grades by a factor giving the best obtained grades in the class close to 36.

Missed exams: Make-up exams will be given to students who have a valid reason for missing an exam, and the reason must be documented. Make-up tests will be more difficult to compensate for the extra study time available to the student.

Unusual attendance and related circumstances: In case of unusual circumstances requiring more than one missed examination, more than one missed homework, or several missed class periods consult with Professor Krylov.

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