ARITHMETIC COMPACTIFICATIONS OF PEL-TYPE SHIMURA VARIETIES — ERRATA

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(1) In (1.2.1.10) and the previous displayed equation: “$[\tau]: F \to \mathbb{Q}_{[\tau]}$” should be “$[\tau]: F \to \mathbb{Q}_{[\tau]}$”.

(2) In Def. 1.2.1.21, “integrable $O$-lattice” should be simply “$O$-lattice”.

(3) In paragraph 1 of the proof of Prop. 1.2.2.3, “$\text{Sym}_{\varphi}(L_1, L_2) \otimes \mathbb{Z}_p$” should be “$\text{Sym}_{\varphi}(L_1, L_2) \otimes \mathbb{Z}_p$”.

(4) In the proof of Cor. 1.3.1.6, “$h(m_A(x_1, x_2)) = m_G(h(x_1), f(x_2))$” should be “$h(m_A(x_1, x_2)) = m_G(h(x_1), h(x_2))$”.

(5) In Def. 1.3.2.1, “for some $M$ over $S$” should be “for some $M$ over $T$”.

(6) In Def. 1.3.2.19, $N$ should be a section of $(\mathbb{Z}_{>0})_S$ (rather than a global constant).

(7) In 5. of Def. 1.4.2.1, “rational principle level-$\mathcal{H}$-structure” should be “rational level-$\mathcal{H}$-structure”.

(8) In the proof of Lem. 2.1.1.1, “$u := u \otimes R$” should be only “$u \otimes R$”.

(9) In paragraph 3 of the proof of Prop. 2.1.2.2, towards the end, “$\text{Aut}_{\tilde{S}}(\tilde{U}_\alpha | U_{\alpha\beta}, S)$” should be “$\text{Aut}_{\tilde{S}}(\tilde{U}_\alpha | U_{\alpha\beta}, S)$”.

(10) In 3. of Prop. 2.1.3.2, “$o(f; m_{\tilde{X}} + \tilde{X}, m_{\tilde{Y}} + \tilde{Y}, S \to \tilde{S}) = o(f; \tilde{X}, \tilde{Y}, S \to \tilde{S})$” should be “$o(f; m_{\tilde{X}} + \tilde{X}, m_{\tilde{Y}} + \tilde{Y}, S \to \tilde{S}) = o(f; \tilde{X}, \tilde{Y}, S \to \tilde{S})$”.

(11) In the proof of Prop. 2.1.3.2:

(a) In paragraph 1, “By smoothness of $f$” should be “By smoothness of $\tilde{Y}$”.

(b) In paragraph 4, “$c_{\alpha\beta} = c_{\alpha\beta} - df(m_{\tilde{X},\alpha\beta} + f^*(m_{\tilde{Y},\alpha\beta})$” should be “$c_{\alpha\beta} = c_{\alpha\beta} + df(m_{\tilde{X},\alpha\beta} - f^*(m_{\tilde{Y},\alpha\beta})$”.

(12) In the second displayed equation of Cor. 2.1.4.4, the left-hand side should be “$H^1(X, f^*\text{Der}_{X/T} \otimes \mathcal{F})$”.

(13) In the last paragraph of the proof of Prop. 2.1.5.3, “$\xi_{\alpha\beta}^* (\tilde{l}_{\alpha\beta})$ and $(\xi_{\alpha\beta}^*)^*(\tilde{l}_{\alpha\beta})$ become the same $l_{\alpha\beta}$ modulo $\mathcal{F}$” should be “$\xi_{\alpha\beta}^* (\tilde{l}_{\beta\gamma})$ and $(\xi_{\alpha\beta}^*)^*(\tilde{l}_{\beta\gamma})$ become the same $l_{\beta\gamma}$ modulo $\mathcal{F}$”.

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(14) In Cor. 2.1.5.15, the \( \text{Lie}_{\mathcal{X}/S} \otimes \text{Lie}_{\mathcal{X}/S} \) in the commutative diagram should be \( \text{Lie}_{\mathcal{X}/S} \otimes \text{Lie}_{\mathcal{X}/S} \).

(15) At the end of paragraph 2 of Sec. 2.1.6, and in the last paragraph preceding Prop. 2.1.6.1, \( R^i \pi_* \Omega^i_{U_{\alpha}} / S \) is trivial for all \( i > 0 \) should be \( R^i \pi_* \Omega^i_{U_{\alpha}} / S \) is trivial for all \( i > 0 \) and all \( q \).

(16) In paragraph 3 of Sec. 2.1.6, in the first two displayed equations, the indices should start with \( \alpha_0 \) instead of \( \alpha_1 \).

(17) In paragraph 3 of the proof of Prop. 2.1.6.4, \( x_{\alpha\beta}^{(1,0)} := f^*_{\alpha}(y_{\alpha\beta}^{(1,0)}) + T_{\alpha\beta}(y_{\alpha\beta}^{(1,0)}) \) should be \( x_{\alpha\beta}^{(1,0)} := f^*_{\alpha}(y_{\alpha\beta}^{(1,0)}) + T_{\alpha\beta}(y_{\alpha\beta}^{(1,0)}) \).

(18) In the proof of Prop. 2.2.2.5, paragraph 2, in the last sentence, \( \tilde{g} \circ j_1 = \text{Id}_A \) and \( \tilde{g} \circ j_2 = \hat{e} \circ \pi \) do lift the morphisms \( g \circ j_1 \) and \( g \circ j_2 \) should be \( \text{Id}_A \) and \( \hat{e} \circ \pi \) do lift the morphisms \( g \circ j_1 = \text{Id}_A \) and \( g \circ j_2 = e \circ \pi \).

(19) In the proof of Prop. 2.2.2.5, paragraph 3, in the displayed equation, \( \text{pr}_2^* H^1(A_0, \mathcal{O}_{A_0}) \) should be \( \text{pr}_2^* H^1(A_0, \mathcal{O}_{A_0}) \); after the displayed equation, \( \text{the pullback from one of the two factors pr}_2^* H^1(A_0 \times A_0, g_0^!((\operatorname{Def}_{A_0})_{k/S}) \otimes T) \) should be \( \text{a sum of elements of the two factors pr}_2^* H^1(A_0, \mathcal{O}_{A_0}) \otimes \mathcal{O}_{A_0/S} \).

(20) In the paragraph following (2.2.3.6), \( [(A_R, f_{0,R})] \in \text{Def}_{A_0}(q)^{-1}([(A_R, f_{0,R})]) \) should be \( [(A_R, f_{0,R})] \in \text{Def}_{A_0}(q)^{-1}([(A_R, f_{0,R})]) \).

(21) In the first displayed equation after (2.2.3.8), \( \text{Def}_{A_0}(p)([(A_R, \lambda_R, f_{0,R})]) \) should be \( \text{Def}_{A_0}(p)([(A_R, \lambda_R, f_{0,R})]) \).

(22) In paragraph 2 of the proof of Prop. 2.2.4.1, \( j_2(x) = (x, x) \) should be \( j_2(x) = (e, x) \).

(23) In the proof of Thm. 3.3.2.4, the reference \( [61, X] \) should be \( [61, IX] \).

(24) In paragraph 2 of Section 4.3.3, \( \text{adic injections} \) should be \( \text{continuous injections} \).

(25) In paragraph 1 after Rem. 4.5.5.4, \( \text{adic injections} \) should be \( \text{continuous injections} \).

(26) In paragraph 4 of the proof of Prop. 5.1.2.4, the parenthetical remark \( "\text{which is the restriction of the complex conjugation under any homomorphism } \Omega_F \leftrightarrow F \stackrel{\tau}{\hookrightarrow} \mathbb{C} \) should be \( "\text{which is compatible with the complex conjugation under any homomorphism } \Omega_F \leftrightarrow F \stackrel{\tau}{\hookrightarrow} \mathbb{C} \)."

(27) In paragraph 4 of the proof of Prop. 5.2.3.9, in the second last line, \( \text{Hom}(N, Z) \) should be \( \text{Hom}\mathcal{O}(N, Z) \).

(28) In Def. 5.4.2.6, should first define \( M^\Phi_H^\mathcal{X} \) to be the quotient of \( \bigsqcup M^\mathcal{X}_{\eta} \) by \( H_n \), where the disjoint union is over representatives \( (Z_n, \Phi_n, \delta_n) \) (with the same \( (X, Y, \phi) \) in \( (Z_n, \Phi_n, \delta_n) \)), and then define \( M^\mathcal{X}_{\mathcal{H}} \) to be the (finite \( \mathcal{X} \)-étale) quotient of \( M^\Phi_H^\mathcal{X} \) by the subgroup of \( \Gamma_\Phi \) stabilizing \( \Phi_H \) (which is called \( \Gamma_{\Phi, H} \) later in Def. 6.2.4.1). (See below for the precise places for
M^\Phi_H to be used. Also, the previously definition of M^{\Phi_H} as a moduli only for the abelian parts was not useful and should be abandoned.)

(29) In Def. 5.4.2.8, should replace the rather discrete object \((\varphi_{-2, H}, \varphi_0, H)\) in
\[ \alpha^2_H = (\mathbb{Z}_H, \varphi_{-2, H}, \varphi_{-1, H}, \varphi_{0, H}, \delta_H, c_H, c_H^{\vee}, \tau_H) \]
with a subscheme \((\varphi_{-2, H}, \varphi_0, H)\) of \((\varphi_{-2, H}, \varphi_{0, H}) < \times \varphi_{1, H}, \) where \((\varphi_{-2, H}, \varphi_0, H)\) is (indeed a discrete object) as in Def. 5.4.2.1 above, and where \((\varphi_{-2, H}, \varphi_0, H)\) is an étale-locally-defined \( H_n\)-orbits which surjects under the two projections to the orbits defining \((\varphi_{-2, H}, \varphi_0, H)\) and \(-\varphi_{1, H}\). In this case we say that \((\varphi_{-2, H}, \varphi_0, H)\) is induced by \((\varphi_{-2, H}, \varphi_0, H)\). (Then, by the universal property of \( M^{\Phi_H} \) because of its very construction, the torus part \((\mathbb{Z}_H, \Phi_H^\vee) = (X, Y, \phi, \varphi_{-2, H}, \varphi_0, H), \delta_H\) and abelian part \((A, \lambda, i_A, \varphi_{-1, H})\) of \((A, \lambda, i_A, \varphi_{-1, H})\) (\(A, \lambda, i_A, \varphi_{-1, H}, \) respectively. In the paragraph following Lem. 6.2.5.23, “formally étale” should be “étale” (i.e., formally étale and of finite type).

(30) In Lem. 5.4.2.10, the \((\varphi_{-2, H}, \varphi_0, H)\) in the second displayed object should be denoted \((\varphi_{-2, H}, \varphi_0, H)\), and it should be added in the sentence that \((\varphi_{-2, H}, \varphi_0, H)\) induces the \((\varphi_{-2, H}, \varphi_0, H)\) in the given \( \Phi_H \). (See (29) above.) It should be clarified that the assertion of uniqueness up to isomorphism allows isomorphisms inducing automorphisms of \((X, Y, \phi, \varphi_{-2, H}, \varphi_0, H)\).

(31) In Prop. 5.4.3.8 and Def. 5.4.3.9, “\( \mathcal{H} \subset \mathcal{H}^* \)” should be “\( \mathcal{H} \) and \( \mathcal{H}^* \)”.

(32) In the second last paragraph of Section 6.2.1, “formally étale” should be “étale” (i.e., formally étale and of finite type).

(33) In the paragraph preceding Def. 6.2.5.23, “formally étale” should be “étale” (i.e., formally étale and of finite type).

(34) In the second paragraph of Section 6.2.4, the wording should be changed to reflect the changes made in Def. 5.4.2.8.

(35) In Section 6.2.4, and the construction for general levels is not correctly deduced from the construction for principle levels.

In the displayed equation preceding (6.2.4.3), the definition \( H_n, G^\text{ess}_{h, Z_n} < \times U^\text{ess}_{Z_n} := H_n, G^\text{ess}_{h, Z_n} < \times H_n, U^\text{ess}_{Z_n} \) is wrong. It should be following Def. 5.3.1.11 faithfully by viewing the semidirect product \( G^\text{ess}_{h, Z_n} < \times U^\text{ess}_{Z_n} \) as a subgroup of \( G^\text{ess}(\mathbb{Z}/n\mathbb{Z}) \). (And later \( G^\text{ess}_{h, Z_n} < \times U^\text{ess}_{Z_n} = (G^\text{ess}_{h, Z_n} < \times U^\text{ess}_{Z_n})/U^\text{ess}_{Z_n} \) should be viewed as a subquotient.) In Lem. 6.2.4.6, should consider \( M^{\Phi_H} \) in \( H_n, G^\text{ess}_{h, Z_n} \) instead of \( M^Z \) in \( H_n, G^\text{ess}_{h, Z_n} \), respectively. In the paragraph following Lem. 6.2.4.6, the \( Z^\text{ess}_{Z_n} \) and \( Z^\text{ess}_{Z_n}/U^\text{ess}_{Z_n} \) should be \( G^\text{ess}_{h, Z_n} < \times U^\text{ess}_{Z_n} \) and \( G^\text{ess}_{h, Z_n} < \times U^\text{ess}_{Z_n} \), respectively. As a result, the image \( H_n, G^\text{ess}_{h, Z_n} \) of \( H_n, G^\text{ess}_{h, Z_n} < \times U^\text{ess}_{Z_n} \) in \( G^\text{ess}_{h, Z_n} \) might be smaller than \( H_n, G^\text{ess}_{h, Z_n} \) in general. Hence, in the bottom-right vertical arrow in (6.2.4.3), the quotient \( M^Z / H_n, G^\text{ess}_{h, Z_n} \) should be replaced with \( M^Z / H_n, G^\text{ess}_{h, Z_n} \).
In Prop. 6.2.4.7 and the remainder of Ch. 6, the morphism \( C_{\Phi_H, \delta H} \to M_{\mathcal{H}}^H \) should be replaced with \( C_{\Phi_H, \delta H} \to M_{\mathcal{H}}^H \). (See (28) above.) The latter is an abelian scheme torsor, not exactly an abelian scheme. We should define \( \Xi_{\Phi_H, \delta H} \to C_{\Phi_H, \delta H} \to M_{\mathcal{H}}^H \) as the equivariant quotient of \( \prod \mathcal{H} \Phi_n, \delta_n \to \prod \mathcal{H} \Phi_n, \delta_n \to \prod \mathcal{H} M_{\mathcal{H}}^H \) by \( H_n \), where the disjoint unions are over representatives \( (\mathcal{Z}_n, \Phi_n, \delta_n) \) (with the same \( (X, Y, \phi) \)) in \( (\mathcal{Z}_H, \Phi_H, \delta_H) \), which carries compatible actions of \( \Gamma_{\Phi_H} \). (By construction, \( M_{\mathcal{H}}^H = M_{\mathcal{H}}^Z_H \) when, for some (and hence every) choice of a representative \( (\mathcal{Z}_n, \Phi_n, \delta_n) \) in \( (\mathcal{Z}_H, \Phi_H, \delta_H) \), the image of \( H_n, G_{n, h, \mathcal{Z}_n} \times U_{1, \mathcal{Z}_n} \sim \mathcal{G}_{n, h, \mathcal{Z}_n} \) is \( H_n, G_{n, h, \mathcal{Z}_n} \); i.e., when the image of \( H_n, G_{n, h, \mathcal{Z}_n} \times \mathcal{G}_{n, h, \mathcal{Z}_n} \) is the direct product \( H_n, G_{n, h, \mathcal{Z}_n} \times H_n, G_{n, h, \mathcal{Z}_n} \); the abelian scheme torsor \( C_{\Phi_H, \delta H} \to M_{\mathcal{H}}^H \) is an abelian scheme when, for some (and hence every) choice of a representative \( (\mathcal{Z}_n, \Phi_n, \delta_n) \) in \( (\mathcal{Z}_H, \Phi_H, \delta_H) \), the splitting of the canonical homomorphism \( C_{\mathcal{H}, h, \mathcal{Z}_n} \times U_{1, \mathcal{Z}_n} \sim \mathcal{G}_{\mathcal{H}, h, \mathcal{Z}_n} \) defined by \( \delta_n \) induces a splitting of the canonical homomorphism \( H_n, G_{n, h, \mathcal{Z}_n} \times U_{1, \mathcal{Z}_n} \sim H_n, G_{n, h, \mathcal{Z}_n} \), and hence an isomorphism \( H_n, G_{n, h, \mathcal{Z}_n} \times U_{1, \mathcal{Z}_n} \sim \mathcal{G}_{n, h, \mathcal{Z}_n} \). It should be noted that, by definition, \( \Gamma_{\Phi_H} \) acts compatibly on \( C_{\Phi_H, \delta H} \) and \( M_{\mathcal{H}}^H \), but trivially on \( M_{\mathcal{H}}^Z_H \), and the canonical morphism \( M_{\mathcal{H}}^H \to M_{\mathcal{H}}^Z_H \) induces a canonical isomorphism \( M_{\mathcal{H}}^H / \Gamma_{\Phi_H} \to M_{\mathcal{H}}^Z_H \). (6.2.4.8), “\( \overline{\text{Pic}}(C_{\Phi_H, \delta H} / M_{\mathcal{H}}^Z_H) \)” should be “\( \overline{\text{Pic}}(C_{\Phi_H, \delta H}) \)” in the proof of Prop. 6.2.5.18, when computing the sheaves of differentials by applying Prop. 2.3.5.2, it is harmless to replace \( M_{Z_H}^H \) with \( M_{\mathcal{H}}^H \) because \( M_{\mathcal{H}}^H \) is finite étale over \( M_{Z_H}^H \).

(36) In Step 1 of Construction 6.3.1.1, the \((\varphi_{-2, H}, \varphi_{0, H})\) in the second displayed object should be denoted \((\varphi_{-2, H}, \check{\varphi}_{0, H})\), and it should be added in the sentence that \((\varphi_{-2, H}, \check{\varphi}_{0, H})\) induces the \((\varphi_{-2, H}, \varphi_{0, H})\) in the given \( \Phi_{\mathcal{H}} \). (See (29) above.) Similarly, the \((\varphi_{-2, H}, \check{\varphi}_{0, H})\) in the second last displayed object should be denoted \((\varphi_{-2, H}, \check{\varphi}_{0, H})\), and it should be added in the sentence that \((\varphi_{-2, H}, \check{\varphi}_{0, H})\) induces the \((\varphi_{-2, H}, \varphi_{0, H})\) in \( \Phi_{\mathcal{H}} \). The uniqueness of the objects in \( D_{\text{PEL}, M_{\mathcal{H}}}^{\text{fil.-spl.}}(R) \) or \( D_{\text{PEL}, M_{\mathcal{H}}}^{\text{fil.-spl.}}(R^\dagger) \) are only up to isomorphism inducing automorphisms on \( \Phi_{\mathcal{H}} \) or \( \check{\Phi}_{\mathcal{H}} \).

(37) In the displayed object after (6.2.4.2), the \((\varphi_{-2, H}, \varphi_{0, H})\) should be denoted \((\varphi_{-2, H}, \check{\varphi}_{0, H})\), and it should be added in the sentence that \((\varphi_{-2, H}, \check{\varphi}_{0, H})\) induces the \((\varphi_{-2, H}, \varphi_{0, H})\) in \( \Phi_{\mathcal{H}} \). (See (29) above.)

(38) In the definition of (6.2.5.9), the invertible sheaf \( \Psi_{\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}}^{\ell} \) does not have to be rigidified. (Hence it is harmless to replace “\( \overline{\text{Pic}}(C_{\Phi_{\mathcal{H}}, \delta_{\mathcal{H}} / M_{\mathcal{H}}^H}) \)” with “\( \overline{\text{Pic}}(C_{\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}}) \)” in the correction of (6.2.4.8) above.)

(39) In proof of Prop. 6.2.5.18, all instances of “\( \Omega_{\mathcal{Z}_n, \delta_n / S_0}^{\Phi_n, \delta_n} [d \log \infty] \)” should be “\( \Omega_{\mathcal{Z}_n, \delta_n / S_0}^{\Phi_n, \delta_n} [d \log \infty] \)”, and all instances of “\( \Omega_{\mathcal{Z}_n, \delta_n / C_{\Phi_n, \delta_n}}^{\Phi_n, \delta_n} [d \log \infty] \)” should be “\( \Omega_{\mathcal{Z}_n, \delta_n / C_{\Phi_n, \delta_n}}^{\Phi_n, \delta_n} [d \log \infty] \).”
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(40) In (6.2.5.22), the $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ should be denoted $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$, and it should be remarked that $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ induces the $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in $\Phi_{\mathcal{H}}$. (See (29) above.)

(41) In Prop. 6.2.6.7, line -2, “over $M_{\mathcal{H}}^C$” should be dropped.

(42) In Prop. 6.3.1.6, “formally étale” should be “étale” (i.e., formally étale and of finite type).

(43) In 6. of Prop. 6.3.1.6, in the second paragraph, the $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in $\alpha^1_{\mathcal{H}} = (Z_{\mathcal{H}}, \varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}}, \delta_{\mathcal{H}}, c_{\mathcal{H}}, e_{\mathcal{H}}, \tau_{\mathcal{H}})$ should be denoted $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$, and it should be added in the sentence that $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ induces the $(\varphi_{-2,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ in $\Phi_{\mathcal{H}}$. (See (29) above.) Also, “Let $(Z_{\mathcal{H}}, \Phi_{\mathcal{H}}, \delta_{\mathcal{H}})$ be a representative of this cusp label” should be “Suppose $(Z_{\mathcal{H}}, \Phi_{\mathcal{H}}, \delta_{\mathcal{H}})$ is a representative of this cusp label”.

(44) In Cor. 6.3.1.8, should assume that $S_{\text{for}}$ is noetherian.

(45) In Cor. 6.3.1.14, should assume that $k$ is of finite type over $\bar{k}$.

(46) In Cor. 6.3.1.18, should assume that $f$ induces an isomorphism between separable closures of residue fields.

(47) In the proof of Prop. 6.3.2.1, should denote the $\Phi_{\mathcal{H}_i}$ in the two displayed degeneration data by two different notations (other than the prescribed $\Phi_{\mathcal{H}_i}$), and remark that they can be approximated because they are discrete in nature. (See (29) above.)

(48) In Rem. 6.3.2.8, “(◦\bar{\mathcal{G}}, ◦\lambda, ◦\check{i}, ◦\alpha_{\mathcal{H}})” should be “(G, λ, i, α_{\mathcal{H}})”.

(49) In the proof of Prop. 6.3.3.11, should use both $(A^\dagger, \lambda_{A^1}, i_{A^1}, \varphi_{-1,\mathcal{H}})$ and the $\Gamma_{\Phi_{\mathcal{H}_{\dagger}}}$-orbit of $\Phi_{\mathcal{H}_{\dagger}} = (X^\dagger, Y^\dagger, \phi^\dagger, \varphi_{-1,\mathcal{H}}, \varphi_{0,\mathcal{H}})$ to determine a morphism $\text{Spec}(R) \to M_{\mathcal{H}_{\dagger}}^{\dagger}$. 

(50) In step 2 of the proof of Prop. 6.3.3.13, for $i = 1, 2$, the $(\varphi_{-2,\mathcal{H}_{i}}, \varphi_{0,\mathcal{H}_{i}})$ in $\alpha^1_{\mathcal{H}_{i}}$ should be denoted $(\varphi_{-2,\mathcal{H}_{i}}, \varphi_{0,\mathcal{H}_{i}})$, and it should be added in the sentence that $(\varphi_{-2,\mathcal{H}_{i}}, \varphi_{0,\mathcal{H}_{i}})$ induces the $(\varphi_{-2,\mathcal{H}_{i}}, \varphi_{0,\mathcal{H}_{i}})$ in $\Phi_{\mathcal{H}_{i}}$. (See (29) above.)

(51) In Rem. 6.3.3.16, “descends” should be “descend”.

(52) In 2. of Thm. 6.4.1.1, $\mathcal{X}_{\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}, \sigma}$ is incorrectly described. It should be “$\mathcal{X}_{\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}, \sigma}$ (before quotient by $\Gamma_{\Phi_{\mathcal{H}}, \sigma}$) admits a canonical structure as the completion of an affine toroidal embedding $\Xi_{\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}, \sigma}$ (along its $\sigma$-stratum $\Xi_{\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}, \sigma}$) of a torus $\Xi_{\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}}$ over an abelian scheme torsor $C_{\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}}$ over a finite étale cover $M_{\mathcal{H}_{\dagger}}^{\dagger}$ of the algebraic stack $M_{\mathcal{H}_{\dagger}}^{\dagger}$.

(53) In 5. of Thm. 6.4.1.1, “formally étale” should be “étale” (i.e., formally étale and of finite type). In the corresponding paragraph of the proof, the first instance of “formally étale” (in the parenthetical remark) should be “étale”, while the second instance can be removed harmlessly.

(54) In 6. of Thm. 6.4.1.1, the $\lambda^+_{A^1}$ and $i^+_{A^1}$ should be denoted $\lambda_{A^1}$ and $i_{A^1}$; just to clarify, the condition for $\varphi^\dagger$ to contain all $v \in B^\dagger$ means for all $v$ centered at the same given geometric point $s$.

(55) In paragraph 4 of the proof of Thm. 6.4.1.1: “(\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}, \sigma)$-stratum” should be “[(\Phi_{\mathcal{H}}, \delta_{\mathcal{H}}, \sigma)]-stratum”. 

(56) In paragraph 6 of the proof of Thm. 6.4.1.1: “formally étale” should be “étale” (i.e., formally étale and of finite type). In the corresponding paragraph of the proof, the first instance of “formally étale” (in the parenthetical remark) should be “étale”, while the second instance can be removed harmlessly.
In Section 7.1.2, paragraph 1, it is a mistake to call $M_H^{Z}$ a “moduli scheme” because it is not necessarily a scheme.

In the proof of Lem. 7.1.2.13 and its proof, should remark that it is constant along the fibers because it is also invariant under $\Gamma_{\Phi_H}$, and we know $M^{\Phi_H}_H / \Gamma_{\Phi_H} \cong M_{H}^{Z}$.

In the proof of Cor. 7.2.3.11, “$M^{\Phi_H}_H |_{Z[\{\Phi_H, \delta_H, \sigma_H\}]} : Z[\{\Phi_H, \delta_H, \sigma_H\}] \rightarrow Z[\{\Phi_H, \delta_H, \sigma_H\}]$” should be “$f_H |_{Z[\{\Phi_H, \delta_H, \sigma_H\}]} : Z[\{\Phi_H, \delta_H, \sigma_H\}] \rightarrow Z[\{\Phi_H, \delta_H, \sigma_H\}]$”.

In the proof of 3. of Prop. 7.2.4.3, the assertion that “the action of $\Gamma'_{\Phi_H} / \Gamma_{\Phi_H}$ on $M^{\Phi_H}_H$ makes $M^{\Phi_H}_H \rightarrow M_{H}^{Z}$ an étale $(\Gamma_{\Phi_H} / \Gamma'_{\Phi_H})$-torsor. For each $\ell_0$ in $P'_H$, consider its stabilizer $\Gamma'_{\Phi_H, \ell_0}$ in $\Gamma'_{\Phi_H}$. Then the correct statement is that the formation of $\Gamma'_{\Phi_H, \ell_0}$-invariants in $(E^{f_0}_{\Phi_H, \delta_H})^{\lambda}_{x}$ commutes with the base change from $S$ to $s$ under the assumption that the condition (7.2.4.6) is satisfied. (When $H$ is neat, it can be shown that $\Gamma'_{\Phi_H, \ell_0}$ acts trivially on $(E^{f_0}_{\Phi_H, \delta_H})^{\lambda}_{x}$.) The details are supplied in a revision.

In Prop. 7.2.3.2, “$\phi' \rightarrow (f')^{-1}T' \cdot \mathcal{O}_{\tilde{W}}$” should be “$(f')^{-1}T' \cdot \mathcal{O}_{\tilde{W}} \rightarrow \chi$”, and “$\phi'' \rightarrow f^{-1}T(d_0) \cdot \mathcal{O}_{\tilde{W}}$” should be “$f^{-1}T(d_0) \cdot \mathcal{O}_{\tilde{W}} \rightarrow \phi''$”.

In 4. of Def. 7.3.1.1, “$x, y \in S_{\Phi_H}$” should be “$x, y \in P_{\Phi_H}$”.

In Lem. 7.3.1.7, “$\mathcal{K}_{\rho_0, \Phi_H}$” should be “$\mathcal{K}_{\rho_0, \Phi_H}$”.

The literal statements of Lem. 7.3.1.9, which we cited almost verbatim from Faltings–Chai (Ch. V, Lem. 5.5), are unfortunately incorrect. For example, if $P_{\Phi_H} = \mathbb{R}_{>0} = \sigma$, then there are no other top-dimensional cones at all, and hence the lemma asserts that $\sigma' = \{0\}$—but $\sigma'$ is certainly nonzero. This error was inherited from a similar error in Ash–Mumford–Rapoport–Tai (Ch. IV, Sec. 2, p. 330). To fix this:

(a) Rewrite the statements of Lem. 7.3.1.9 as follows: “Suppose $\sigma \in \Sigma_{\Phi_H}$, and suppose $\sigma_1, \ldots, \sigma_r$ are the one-dimensional faces of $\sigma$.
each $1 \leq j \leq r$, consider the unique $y_j \in \sigma_j$ such that $S_{\Phi H}^j \cap \sigma_j = \mathbb{Z}_{\geq 1} \cdot y_j$, so that $K_{\text{pol}_{\Phi H}} \cap \sigma_j = \mathbb{R}_{\geq 1} \cdot (\text{pol}_{\Phi H}(y_j))^{-1} \cdot y_j$, and let $L_j := \{ x \in S_{\Phi H} \otimes \mathbb{R} : \langle x, y_j \rangle = \text{pol}_{\Phi H}(y_j) \}$. Then each $L_j \cap K_{\text{pol}_{\Phi H}}$ is a top-dimensional face of $K_{\text{pol}_{\Phi H}}^\vee$, whose vertices are in $S_{\Phi H} \cap K_{\text{pol}_{\Phi H}}^\vee$, because $y_j \in S_{\Phi H} \cap \text{pol}_{\Phi H}$, and pol$_{\Phi H}$ takes integral values on $S_{\Phi H}^j$, and the intersection $\bigcap_{1 \leq j \leq r} (L_j \cap K_{\text{pol}_{\Phi H}}^\vee)$ defines a face of $K_{\text{pol}_{\Phi H}}^\vee$ (which we consider dual to $\sigma$). Suppose $d \geq 1$ is any integer, and suppose $\ell_0 \in S_{\Phi H} \cap d \cdot (\bigcap_{1 \leq j \leq r} (L_j \cap K_{\text{pol}_{\Phi H}}^\vee))$ does not lie on any proper face of $d \cdot (\bigcap_{1 \leq j \leq r} (L_j \cap K_{\text{pol}_{\Phi H}}^\vee))$. Then there exist $\ell_1, \ldots, \ell_n \in S_{\Phi H} \cap K_{\text{pol}_{\Phi H}}^\vee$ (which are not necessarily vertices of $K_{\text{pol}_{\Phi H}}^\vee$) such that $\mathbb{R}_{\geq 0} \cdot \sigma^\vee = \sum_{1 \leq i \leq n} \mathbb{R}_{\geq 0} \cdot (d \cdot \ell_0)$. By (7.3.3.6)

(with $\ell_{\text{gen}} = \ell_i$ there, for each $i$), we see that $v(\Psi_{\Phi H, \delta H}(\ell)) \geq 0$ for all $\ell \in \tau^\vee$.”

(69) In Def. 7.3.3.1, “$\mathbb{Z}_{\geq 0}$-generator” should be “$\mathbb{Z}_{\geq 0}$-generator”.

(70) In Thm. 7.3.3.4, 1., and in its proof, all instances of “$\otimes$ $d_0 \cdot \mathbf{f}_{\text{H.pol}}$” should be “$\otimes$ $d_0 \cdot \mathbf{f}_{\text{H.pol}}$”, “$\otimes$ $d_0 \cdot \mathbf{f}_{\text{H.pol}}$” should be “$\otimes$ $d_0 \cdot \mathbf{f}_{\text{H.pol}}$”. (71) In the proof of 1 of Thm. 7.3.3.4:

(a) In Paragraph 2, “$\mathbb{Z}_{\geq 0}$-generator” should be “$\mathbb{Z}_{\geq 0}$-generator.”

(b) In Paragraph 4, “ample line bundle $\otimes_{1 \leq i \leq r} (\text{pr}_i^* (\text{Id}_A, \lambda_A)^* \mathcal{P}_A) \otimes e_i$,” should be just “line bundle $\otimes_{1 \leq i \leq r} (\text{pr}_i^* (\text{Id}_A, \lambda_A)^* \mathcal{P}_A) \otimes e_i$”, the various instances of “$\text{Hom}$” should be “$\text{Hom}$”, and “$\Psi_{\Phi H, \delta H}(\ell_0)$ is ample over $M_{\text{H}}^{\mathbb{Z}_{\geq 0}}$” should be “$\Psi_{\Phi H, \delta H}(\ell_0)$ is relatively ample over $M_{\Phi H}^{\mathbb{Z}_{\geq 0}}$.”

(c) In Paragraph 5, “$(d \cdot \ell_0 + \tau^\vee) / \Gamma_{\Phi H}$” should be “$(d \cdot \ell_0 + \tau^\vee) / \Gamma_{\Phi H}$.”
(d) In paragraph 6, “structural sheaf of $\oplus_{\ell \in T^\nu} (\Psi_{\Phi_H, \delta_H}(\ell))^{\wedge}_{\hat{x}}$” should be “$\mathcal{O}_{X_{\Phi_H, \delta_H, T}} \cong \oplus_{\ell \in T^\nu} (\Psi_{\Phi_H, \delta_H}(\ell))^{\wedge}_{\hat{x}}$”

(72) In the proof of 2 of Thm. 7.3.3.4:

(a) In paragraph 2, the morphism $y : \text{Spf}(V) \to M^\nu_H$ should be required to induce morphisms $\text{Spec}(V) \to \Xi_{\Phi_H, \delta_H}(\sigma)$ and $\text{Spec}(V) \to M^\nu_H$ mapping the generic point of $\text{Spec}(V)$ to $\Xi_{\Phi_H, \delta_H}$ and $M_H$, respectively. Also, “$y$ is uniquely determined by $\hat{y}$” should be “there are only finitely many $\hat{y}$ inducing the same $\hat{z}$”.

(b) In (7.3.3.5), “$\mathcal{X}_{\Phi_H, \delta_H, \sigma}$” should be “$(\mathcal{X}_{\Phi_H, \delta_H, \sigma})^{\wedge}_{\hat{z}}$”.

(c) In paragraph 5, all instances of “$d_0 \cdot K_{\text{pol}_{\Phi_H}}$” should be “$S_{\Phi_H} \cap (d_0 \cdot K_{\text{pol}_{\Phi_H}})$”.

(d) In the second last paragraph, it is literally incorrect to consider the pullbacks to $(X_{\Phi_H, \delta_H, \sigma})^{\wedge}_{\hat{z}}$ of sections of $\mathcal{O}_{U_f}$ and of the (coherent ideal) pullback of $(\mathcal{J}_{H, \text{pol}}^{(d_0)})^{\wedge}_{\hat{z}}$ to $U_f$. To fix this:

(i) In paragraph 5 (of the proof), add the following sentences: “Without loss of generality, we may and we shall assume that $f(\ell) \neq 0$ exactly when $\ell \in \Gamma_{\Phi_H} \cdot \ell_0$. Let $\mathfrak{M}_{f(\ell_0)}$ denote the maximal open formal subscheme of $(C_{\Phi_H, \delta_H})^{\wedge}_{\hat{z}}$ over which $f(\ell_0)$ is a generator of the pullback of $\Psi_{\Phi_H, \delta_H}(\ell_0)$, and let $\mathfrak{M}_{f(\ell_0)}$ denote the preimage of $\mathfrak{M}_{f(\ell_0)}$ under the canonical morphism $(X_{\Phi_H, \delta_H, \sigma})^{\wedge}_{\hat{z}} \to (C_{\Phi_H, \delta_H})^{\wedge}_{\hat{z}}$. Then the proof of 1 of Theorem 7.3.3.4 shows that $\mathfrak{M}_{f(\ell_0)}$ is the preimage of $U_f$ under the canonical morphism $(X_{\Phi_H, \delta_H, \sigma})^{\wedge}_{\hat{z}} \to \text{Bl}_{(\mathcal{J}_{H, \text{pol}}^{(d_0)})^{\wedge}_{\hat{z}}}((M^\nu_H)^{\wedge}_{\hat{z}})$.”

(ii) In the second last paragraph, “pullback of sections of $\mathcal{O}_{U_f}$ to $(X_{\Phi_H, \delta_H, \sigma})^{\wedge}_{\hat{z}}$” should be “pullback of sections of $\mathcal{O}_{U_f}$ to the open formal subscheme $\mathfrak{M}_{f(\ell_0)}$ of $(X_{\Phi_H, \delta_H, \sigma})^{\wedge}_{\hat{z}}$”, “pullback of sections of $(\mathcal{J}_{H, \text{pol}}^{(d_0)})^{\wedge}_{\hat{z}}$ to $(X_{\Phi_H, \delta_H, \sigma})^{\wedge}_{\hat{z}}$” should be “pullback to $\mathfrak{M}_{f(\ell_0)}$ of sections of the (coherent ideal) pullback of $(\mathcal{J}_{H, \text{pol}}^{(d_0)})^{\wedge}_{\hat{z}}$ to $U_f$”; “pullback of $(\mathcal{J}_{H, \text{pol}}^{(d_0)})^{\wedge}_{\hat{z}}$ to $(X_{\Phi_H, \delta_H, \sigma})^{\wedge}_{\hat{z}}$” should be “pullback of $(\mathcal{J}_{H, \text{pol}}^{(d_0)})^{\wedge}_{\hat{z}}$ to $\mathfrak{M}_{f(\ell_0)}$”; and all instances of “sections in $\oplus_{\ell \in \sigma^\nu} (\Psi_{\Phi_H, \delta_H}(\ell))^{\wedge}_{\hat{x}}$ over $\mathfrak{M}_{f(\ell_0)}$”.

(iii) In the last paragraph, both instances of “$(X_{\Phi_H, \delta_H, \sigma})^{\wedge}_{\hat{z}}$” should be “$\mathfrak{M}_{f(\ell_0)}$”.

(e) In the last paragraph of the proof, both instances of “$(\Psi_{\Phi_H, \delta_H})^{\wedge}_{\hat{x}}$” should be “$(\Psi_{\Phi_H, \delta_H}(\ell))^{\wedge}_{\hat{x}}$”, and should only assert that $\hat{x}$ determines a compatible collection of morphisms $\{\Gamma(\mathfrak{M}_{f(\ell_0)}), (\Psi_{\Phi_H, \delta_H}(\ell))^{\wedge}_{\hat{x}} \to k\}$ for $\ell$’s in a finite index subgroup of $\sigma^\perp$. 
(73) In Def. A.1.2.1, 2., (ii): “for each three objects \( X, Y, Z \in \text{Ob}\ C \)” should be “for each two objects \( X, Y \in \text{Ob}\ C \)”.

(74) In the second paragraph of Def. A.7.2.8, “\( U \to X \)” should be “\( U \to Y \)”.

(75) In 5. of Thm. B.3.7, “of finite type \( S \)” should be “of finite type over \( S \)”.

(76) In the third paragraph of the proof of Thm. B.3.11, “show that \( \xi \) is formally étale” should be “show that \( \tilde{\xi} \) is formally étale”, and “Krull dimension of \( U \) and \( X \)” should be “Krull dimensions of \( X \) and \( X' \)”.

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