COMPARISON BETWEEN ANALYTIC AND ALGEBRAIC CONSTRUCTIONS OF TOROIDAL COMPACTIFICATIONS OF PEL-TYPE SHIMURA VARIETIES — ERRATA

KAI-WEN LAN

(1) In Def. 1.1.2, at the end, should write “for example, when \( L = \{0\} \)”.

(2) In Lem. 3.1.1 and its proof, the references “[4, 1.5]” should be “[4, 3.5]”.

(3) In the last displayed equation preceding Def. 3.1.2, the last “\( F(g) \)” should be “\( F_{g-i} \)”.

(4) In Def. 3.1.2, \( \varphi^{(g)}_- \) and \( \varphi^{(g)}_0 \) should be the inverse isomorphisms of what were literally written.

(5) In Lem. 3.1.6, “at any level \( \mathcal{H} \), the moduli problem defined by
\[
(\text{Gr}_{-1}^{F(g)}, \langle \cdot, \cdot \rangle^{(g)}, (h_0)_{-1})
\]
should better be “at any neat level \( \mathcal{H} \), the scheme \( \mathcal{M}_{\mathcal{H}}^{F(g)} \) can be identified with the moduli problem defined by \( (\text{Gr}_{-1}^{F(g)}, \langle \cdot, \cdot \rangle^{(g)}, (h_0)_{-1}) \) at a suitable level (\( \mathcal{H}'_{-1} \), to be introduced in §3.4 below)”.

(6) Rem. 3.3.1, Lem. 3.3.3, and the diagram following them are correct for principal congruence subgroups, but incorrect in general. This does not affect the main comparisons done for the various structures at principal levels, but is still an unfortunate mistake that has to be corrected. At the same time, there are similar mistakes in [1] and in [2], which we have noticed and fixed. Let us explain how to carry out the corrections (compatible with the corrections made to [1] and in [2] in their errata) and the necessary improvements for the exposition:

(a) In Def. 3.2.1, after defining \( P_{F(g)}(R) \), add the definition
\[
P'_{F(g)}(R) := \{(p, r) \in P(g)(R) : \text{Gr}_{-2}(p) = r \text{Id}_{\text{Gr}_{-2}} \text{ and } \text{Gr}_0(p) = \text{Id}_{\text{Gr}_0}\}.
\]
Then add or modify the natural inclusions and exact sequences in Lem. 3.2.2 accordingly.

(b) In §3.3, add or revise the definitions
\[
\Gamma_{\mathcal{H}}^{F(g),+} := \Gamma_{\mathcal{H}}^{(g)} \cap P_{F(g)}(Q) = (g\mathcal{H}g^{-1}) \cap P'_{F(g)}(Q),
\]
\[
\Gamma_{\mathcal{H}}^{F(g),h,+} := \text{image of } \Gamma_{\mathcal{H}}^{F(g)} \text{ under the homomorphism } P_{F(g)}(Q) \to G_{h,F(g)}(Q),
\]
\[
\Gamma_{\mathcal{H}}^{F(g),h} := \text{image of } \Gamma_{\mathcal{H}}^{F(g),+} \text{ under the homomorphism } P'_{F(g)}(Q) \to G_{h,F(g)}(Q),
\]
\[
\Gamma_{\mathcal{H}}^{F(g),l,+} := \text{image of } \Gamma_{\mathcal{H}}^{F(g)} \text{ under the homomorphism } P_{F(g)}(Q) \to G_{l,F(g)}(Q),
\]
\[
\Gamma_{\mathcal{H}}^{F(g),l} := \text{image of } \Gamma_{\mathcal{H}}^{F(g),+} \text{ under the homomorphism } P'_{F(g)}(Q) \to G_{l,F(g)}(Q),
\]
\[
\Gamma_{\mathcal{H}}^{F(g),l,-} := \text{image of } \Gamma_{\mathcal{H}}^{F(g),+} \text{ under the homomorphism } P_{F(g)}(Q) \to G_{l,F(g)}(Q).
\]

Then add or modify the natural inclusions and exact sequences in Lem. 3.3.2 accordingly.

(c) Lem. 3.3.3 should be

The splitting \( \varepsilon(g) : \text{Gr}F^{(g)} \rightarrow L(g) \) defines an isomorphism

\[
P_{p(s)}(\mathbb{Q})/U_{p(s)}(\mathbb{Q}) \cong G_{l,F^{(s)}}(\mathbb{Q}) \times G_{h,F^{(s)}}(\mathbb{Q})
\]

mapping \( \Gamma^{F^{(s)}}_{U,U_2} / \Gamma^{F^{(s)},l_1}_H \) isomorphically to a subgroup of \( \Gamma^{F^{(s)}},l \times \Gamma^{F^{(s)}},h_+ \) containing \( \Gamma^{F^{(s)}},l_1 \times \Gamma^{F^{(s)}},h \). The two projections then induce isomorphisms \( \Gamma^{F^{(s)}},l_1 / \Gamma^{F^{(s)},l} \cong (\Gamma^{F^{(s)}},U_1 / \Gamma^{F^{(s)},l} \times \Gamma^{F^{(s)},h}) \cong \Gamma^{F^{(s)},h_+ / \Gamma^{F^{(s)},h}} \).

When \( H = \mathcal{U}(n) \), we have \( \Gamma^{F^{(s)}},l_1 / \mathcal{U}(n) = \Gamma^{F^{(s)},h} \) and \( \Gamma^{F^{(s)}},h_+ / \mathcal{U}(n) = \Gamma^{F^{(s)},h} \), and hence the above mapping defines an isomorphism \( \Gamma^{F^{(s)}},l_1 / \mathcal{U}(n) \cong \Gamma^{F^{(s)},h_+} \).

(d) The next diagram should be replaced with the following diagram:

\[
\begin{array}{cccc}
X_2^{(s)} & \xrightarrow{\tau_2} & X_1^{(s)} & \xrightarrow{\pi_1} & X_0^{(s)} \\
\text{quot. by } \Gamma^{F^{(s)}},U_2 & \downarrow & \text{quot. by } \Gamma^{F^{(s)}},U_1 & \downarrow & \text{quot. by } \Gamma^{F^{(s)},h} \\
\Gamma^{F^{(s)}},U_1 \backslash X_2^{(s)} & \rightarrow & \Gamma^{F^{(s)}},U_1 \backslash X_1^{(s)} & \rightarrow & \Gamma^{F^{(s)}},h \backslash X_0^{(s)} \\
\text{quot. by } \Gamma^{F^{(s)},h} & \downarrow & \text{quot. by } \Gamma^{F^{(s)},h} & \downarrow & \text{quot. by } \Gamma^{F^{(s)},h} \\
\Gamma^{F^{(s)},h} \backslash (\Gamma^{F^{(s)},U} \backslash X_2^{(s)}) & \rightarrow & \Gamma^{F^{(s)},h} \backslash (\Gamma^{F^{(s)},U_1} \backslash X_1^{(s)}) & \rightarrow & \Gamma^{F^{(s)},h} \backslash X_0^{(s)} \\
\text{quot. by } \Gamma^{F^{(s)},l} & \downarrow & \text{quot. by } \Gamma^{F^{(s)},l} & \downarrow & \text{quot. by } \Gamma^{F^{(s)},l} \\
\Gamma^{F^{(s)},l} \backslash X_2^{(s)} & \rightarrow & (\Gamma^{F^{(s)},U} / \Gamma^{F^{(s)},l}) \backslash (\Gamma^{F^{(s)},U_1} \backslash X_1^{(s)}) & \rightarrow & \Gamma^{F^{(s)},h} \backslash X_0^{(s)}
\end{array}
\]

The changes are in the last row (and in the definitions of \( \Gamma^{F^{(s)},h} \) and \( \Gamma^{F^{(s)},l} \)). (For the bottom-right vertical arrow, we use the isomorphism \( \Gamma^{F^{(s)},l_1} / \Gamma^{F^{(s)},l} \cong (\Gamma^{F^{(s)},U_1} / \Gamma^{F^{(s)},l}) / (\Gamma^{F^{(s)},l_1} / \Gamma^{F^{(s),l}} \times \Gamma^{F^{(s)},h}) \cong \Gamma^{F^{(s)},h_+ / \Gamma^{F^{(s)},h}} \) in Lemma 3.3.3.)

(e) The title of §3.4 should be “The morphism \( \Gamma^{F^{(s)},h} \backslash X_0^{(s)} \rightarrow \Gamma^{F^{(s)},h_+} \backslash X_0^{(s)} \).” After \( H_{-1} \) is defined, we should also introduce \( H_{-1}'' := \text{Gr}_{-1}((gHg^-1) \cap P_{F^{(s)}}(\mathbb{A}^\infty)) \) and \( H_{-1}' := \text{Gr}_{-1}((gHg^-1) \cap (G_{l,F^{(s)}}(\mathbb{Q}) \times P_{F^{(s)}}(\mathbb{A}^\infty))) \), satisfying \( H_{-1}'' \subset H_{-1} \subset H_{-1}' \) and \( H_{-1}' \subset H_{-1} \subset H_{-1}'' \). Later, all instances of \( \Gamma^{F^{(s)},h} \) should be replaced with \( \Gamma^{F^{(s)},h_+} \), and all instances of \( H_{-1} \) should be replaced with \( H_{-1}' \).

Then we insert the following sentences between “the pullback of the universal family.” and “Then, by the same argument in . . . :”:
Similarly, let $M^{\phi}_{\mathcal{H}}$ be the finite étale covering of $M^{\phi}_{\mathcal{H}}$ classifying the additional structure $(\varphi_{-2,H}, \varphi_{0,H})$ inducing $(\varphi_{-2,H}, \varphi_{0,H})$ and $\varphi_{-1,H}$ (see [1, erratum for Def. 5.4.2.6]), and denote its pullback to $\mathbb{C}$ by $M^{\phi}_{\mathcal{H},C}$. Let $M^{\phi}_{\mathcal{H},C,L \otimes \mathbb{Q}} \to M^{\phi}_{\mathcal{H},C,L \otimes \mathbb{Q}}$ be the pullback of $M^{\phi}_{\mathcal{H}} \to M^{\phi}_{\mathcal{H}}$. Then there is also a tautological pair $(\varphi_{-2,H}, \varphi_{0,H})$ inducing $(\varphi_{-2,H}, \varphi_{0,H})$ and $\varphi_{-1,H}$ over $M^{\phi}_{\mathcal{H}}$.

And we replace the next paragraph with the following:

Since $\Gamma^{\phi, h}_{\mathcal{H}} = \mathcal{H}_{1} \cap G_{h, F(g)}(\mathbb{Q})_0$ and $\Gamma^{\phi, h, +}_{\mathcal{H}} = \mathcal{H}_{1} \cap G_{h, F(g)}(\mathbb{Q})_0$, and since $\Gamma^{\phi, h}_{\mathcal{H}(n)} = \Gamma^{\phi, h}_{\mathcal{H}(n)} = \Gamma^{\phi, h}_{\mathcal{H}(n)}$ for all $n \geq 1$ such that $\mathcal{H}(n) \subset \mathcal{H}$, the construction of $M^{\phi}_{\mathcal{H}}$ as the quotient of $\prod M^{\phi}_{\mathcal{H}}$ (with the disjoint union running over representatives $(z_n, b_n, b_n)$, with the same $(X(g), Y(g), \phi(g))$, in $(\mathcal{H}, \mathcal{H}, \delta)$) by $\mathcal{H}/\mathcal{H}(n)$, and the construction of $M^{\phi}_{\mathcal{H}}$ as a quotient of $M^{\phi}_{\mathcal{H}}$ by $\Gamma_{\mathcal{H}} \cong \Gamma^{\phi, h}_{\mathcal{H}}$, show that

$$\text{Sh}_{\mathcal{H}, 0} := \Gamma^{\phi, h}_{\mathcal{H}} \setminus X_0^{\phi, h, +} \to \text{Sh}_{\mathcal{H}, 0} := \Gamma^{\phi, h, +}_{\mathcal{H}} \setminus X_0^{\phi, h, +}$$

is the pullback of the analytification $M^{\phi}_{\mathcal{H}, \text{an}, L \otimes \mathbb{Q}} \to M^{\phi}_{\mathcal{H}, C, L \otimes \mathbb{Q}}$ of $M^{\phi}_{\mathcal{H}, C, L \otimes \mathbb{Q}} \to M^{\phi}_{\mathcal{H}, C, L \otimes \mathbb{Q}}$ under $\text{Sh}_{\mathcal{H}, 0} \to \text{Sh}_{\mathcal{H}} \cong M^{\phi}_{\mathcal{H}, \text{an}, L \otimes \mathbb{Q}}$. Let us (abusively) denote the pullback of the holomorphic family over $\text{Sh}_{\mathcal{H}}$ by (the same notation)

$$(A_{\text{hol}}, \lambda_{A_{\text{hol}}}, (\varphi_{-1,H, \text{hol}})) \to \text{Sh}_{\mathcal{H}, 0} = \Gamma^{\phi, h, +}_{\mathcal{H}} \setminus X_0^{\phi, h, +},$$

and also by

$$(A_{\text{hol}}, \lambda_{A_{\text{hol}}}, (\varphi_{-1,H, \text{hol}})) \to X_0^{\phi, h, +}$$

the further pullback to $X_0^{\phi, h, +}$. Let us denote by

$$(\varphi_{-2,H, \text{hol}}, (\varphi_{-2,H, \text{hol}}, \varphi_{0,H, \text{hol}})) \to \Gamma^{\phi, h}_{\mathcal{H}} \setminus X_0^{\phi, h, +}$$

the pullback of $(\varphi_{-2,H, \text{hol}}, (\varphi_{-2,H, \text{hol}}, \varphi_{0,H, \text{hol}})) \to M^{\phi}_{\mathcal{H}, \text{an}, L \otimes \mathbb{Q}}$. By construction, over each $h_{-1} \in X_0^{\phi, h, +}$, the pullback $(\varphi_{-2,H, h_{-1}, \text{hol}}, (\varphi_{-2,H, h_{-1}, \text{hol}}, \varphi_{0,H, h_{-1}, \text{hol}}))$ of $(\varphi_{-2,H, \text{hol}}, (\varphi_{-2,H, \text{hol}}, \varphi_{0,H, \text{hol}}))$ is (up to isomorphism) the $\mathcal{H}$-orbit of the canonical tuple $((\varphi_{-2,0}, (\varphi_{-1,0})), \varphi_{-1, h_{-1}, g})$ above the $\mathcal{H}$-orbit $\varphi_{-1, h_{-1}, g}$ of...
For later references, let us define $\mathcal{Sh}_{\Phi, alg}^{g(n)}$ (resp. $\mathcal{Sh}_{\mathcal{F}, alg}^{g(n)}$) to be the connected component of $\mathcal{Sh}_{\Phi, alg}^{g(n)} \cong M_{\Phi, \mathcal{C}, L} \otimes \mathbb{Q}$ (resp. $\mathcal{Sh}_{\mathcal{F}, alg}^{g(n)} \cong M_{\mathcal{F}, \mathcal{C}, L} \otimes \mathbb{Q}$) whose analytification is $\mathcal{Sh}_{\Phi, 0}^{g(n)} = \Gamma_{\Phi, h, \mathcal{X}}^{0}$ (resp. $\mathcal{Sh}_{\mathcal{F}, 0}^{g(n)} = \Gamma_{\mathcal{F}, h, \mathcal{X}}^{0}$).

In Remark 3.4.2, add the following sentence:
Similarly, the fiber-wise description in the paragraph preceding Lemma 3.4.1 determines $(\varphi^{(g)}, \sim_{-2,\mathcal{H}, hol}, \varphi_{0,\mathcal{H}, hol}) \rightarrow \Gamma_{\mathcal{H}, h, \mathcal{X}}^{0}$.

(f) In Lem. 3.5.11, “$C_{\Phi, n, \mathcal{H}, \mathcal{C}, alg} \rightarrow \mathcal{Sh}_{\mathcal{H}, 0, alg}^{g(n)}$” should be “$C_{\Phi, n, \mathcal{H}, \mathcal{C}, alg} \rightarrow \mathcal{Sh}_{\mathcal{H}, 0, alg}^{g(n)}$”.

(g) In Cor. 3.5.12, “$\mathcal{Sh}_{\mathcal{H}, alg}^{g(n)}$” should be replaced with “$\mathcal{Sh}_{\mathcal{H}, alg}^{g(n)}$”, and “$\mathcal{Sh}_{\mathcal{F}, alg}^{g(n)}$” should be replaced with “$\mathcal{Sh}_{\mathcal{F}, alg}^{g(n)}$”.

(h) In §4.2, should also mention the pair $(\varphi^{(g)}, \sim_{-2,\mathcal{H}, \mathcal{F}, 0,\mathcal{H}}, \varphi^{(g)}, \sim_{0,\mathcal{H}})$ (not $(\varphi^{(g)}, \sim_{-2,\mathcal{H}, \mathcal{F}, 0,\mathcal{H}}, \varphi^{(g)}, 0,\mathcal{H})$).

References


Current address: University of Minnesota, Minneapolis, MN 55455, USA
E-mail address: kwan@math.umn.edu