Calculus I Review

Evaluate the Integrals

1. $\int x\sin(x^2)dx$

Solution: $u = x^2$, $du = 2xdx \Rightarrow$

$$\int x\sin(x^2)dx = \frac{1}{2}\int \sin u du = -\frac{1}{2}\cos u + C = -\frac{1}{2}\cos\left(x^2\right) + C$$

2. $\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx$

Solution: $u = \sqrt{x}$, $du = \frac{1}{2}\frac{1}{\sqrt{x}}dx \Rightarrow$

$$\int \frac{\sin\sqrt{x}}{\sqrt{x}}dx = 2\int \sin u du = -2\cos u + C = -2\cos\sqrt{x} + C$$

3. $\int \tan^2\theta\sec^2\theta d\theta$

Solution: $u = \tan\theta$, $du = \sec^2\theta d\theta \Rightarrow$

$$\int \tan^2\theta\sec^2\theta d\theta = \int u^2 du = \frac{1}{3}u^3 + C = \frac{1}{3}\tan^3\theta + C$$

4. $\int e^{\cos t}\sin t dt$

Solution: $u = \cos t$, $du = -\sin t \Rightarrow$

$$\int e^{\cos t}\sin t dt = -\int e^u du = -e^u + C = -e^{\cos t} + C$$

5. $\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}dx$

Solution: $u = \frac{1}{x}$, $du = -\frac{1}{x^2} \Rightarrow$

$$\int_1^2 \frac{e^{\frac{1}{x}}}{x^2}dx = -\int_1^{\frac{1}{2}} e^u du = e - e^{\frac{1}{2}}$$

6. $\int \sin t\sec(\cos t)\tan(\cos t)dt$

Solution: $u = \cos t$, $du = -\sin t dt \Rightarrow$

$$\int \sin t\sec(\cos t)\tan(\cos t)dt = \int \sec u\tan u du = -\sec u + C = -\sec(\cos t) + C$$

7. $\int \frac{1}{x} dx$**.**
Solution: $\ln x + C$
8. $\int \frac{\sin x}{1+\cos^2 x} dx$**.**
Solution: $u = \cos x$**,** $du = -\sin x dx \Rightarrow$

$$\int \frac{\sin x}{1+\cos^2 x} dx = -\int \frac{du}{1+u^2} = -\tan^{-1} u + C = -\tan^{-1}(\cos x) + C$$

9. $\int e^x \sqrt{1+e^x} dx$**.**
Solution: $u = 1 + e^x$**,** $du = e^x dx \Rightarrow$

$$\int e^x \sqrt{1+e^x} dx = \int \sqrt{u} du = \frac{2}{3} u^{\frac{3}{2}} = \frac{2}{3} \left(1+e^x\right)^{\frac{3}{2}}$$