Marta Lewicka, Math 5535, Winter 2005

Homework 1

1. Let f be the affine map: $f(x) = ax + b$, where $a, b \in \mathbf{R}$. Fix $x_0 \in \mathbf{R}$ and find the asymptotics of the orbit $\mathcal{O}(x_0)$. That is, describe the behaviour of $f^{(n)}(x_0)$ as $n \to +\infty$.

2. Let $f : [a, b] \longrightarrow \mathbf{R}$ be a continuous function, such that $[a, b] \subset f([a, b])$. Prove that f must have a fixed point.

3. Find all fixed points and eventual fixed points of the following maps. Determine the stability of the fixed points.
a) $f(x) = 0$ on $\mathbf{R}$,
b) $f(x) = |x - 1|$ on $[0, 2]$,
c) $f(x) = 1 - x^2$ on $[-1, 1]$,
d) $f(x) = 4x(1 - x)$ on $[0, 1]$ (in this last example, it is enough if you describe the set of eventual fixed points, without calculating them).

4. Let the iterated function be continuous.
(i) Prove that if an orbit has exactly one accumulation point then this point must be a fixed point of f.
(ii) Can an orbit have more than one accumulation point?

5. (i) Does the stability of a fixed point imply its asymptotic stability?
(ii) If a fixed point is attracting, must it be asymptotically stable?