

### Homework 11

1. Let  $f : \mathbf{R}^2 \rightarrow \mathbf{R}^2$  be given as:  $f(x, y) = (y/(1 + x^2), x/(2 + 2y^2))$ . Determine stability of the fixed point  $(0, 0)$ .
2. The same question as in problem 1, for the function  $f(x, y) = (2y/(1 + x^2), 2x/(1 + y^2))$ .
3. Elaydi problem 13 page 178.
4. Elaydi problem 15 page 178.
5. Let  $f : \mathbf{R}^2 \rightarrow \mathbf{R}^2$  be given as:  $f(x, y) = (x + x^2 + y^2, y)$ .
  - (i) Calculate  $Df(0, 0)$  and its spectral radius.
  - (ii) Prove that  $(0, 0)$  is an unstable fixed point of  $f$ .
6. Let  $f : \mathbf{R}^2 \rightarrow \mathbf{R}^2$  be given as:  $f(x, y) = (x + x^3y^2, y)$ .
  - (i) Calculate  $Df(0, 0)$  and its spectral radius.
  - (ii) Prove that  $(0, 0)$  is a stable fixed point of  $f$ .
7. Let  $A \in M^{2 \times 2}$ . Define *the norm of A* as:
$$\|A\| = \sup\{\|Ax\|; x \in \mathbf{R}^2, \|x\| \leq 1\}.$$
  - (i) Prove that  $\rho(A, B) := \|A - B\|$  is a metric on  $M^{2 \times 2}$ .
  - (ii) Prove that for every  $x \in \mathbf{R}^2$  there holds:  $\|Ax\| \leq \|A\| \cdot \|x\|$ .