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Homework 3

Conclude the proof of Sharkovsky's theorem by solving problems 1-3. In these problems we assume that $n = p2^m$, where $p \geq 3$ is odd and $m \geq 2$. Assume that a continuous function $f : I \longrightarrow I$ has a cycle of prime period n. Without of loss of generality we may assume that there f has no cycles of prime period $p2^l$, for any $l < m$.

1. Prove that f must have a cycle of prime period $q2^m$, for any q odd and greater than p.

2. Prove that f must have a cycle of prime period 2^k, for any $k \leq m$.

3. Prove that f must have a cycle of prime period $l2^m$, for any even l.

4. For $n \geq 3$ and odd, define the function $f : [0, 1] \longrightarrow [0, 1]$ in the following way.

$$f(0) = 1/2, \qquad f(1/(n+1)) = 1, \qquad f(1/2) = 1/2 + 1/(n+1),$$

$$f(1/2 + 1/(n+1)) = 1/2 - 1/(n+1), \qquad f(1) = 0.$$

On intervals $[0, 1/(n+1)]$, $[1/(n+1), 1/2]$, $[1/2, 1/2+1/(n+1)]$, $[1/2+1/(n+1), 1]$ the function f is linear. Show that f has a cycle with prime period $n + 2$ but not a cycle with prime period n.

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6. Let $f : I \longrightarrow I$ be a continuous function. Show that if there exists a point $x \in I$ such that $f^{(2)}(x) < x < f(x)$, or $f(x) < x < f^{(2)}(x)$, then f must have a fixed point. Must f have a cycle of prime period 2 as well?