

### Homework 5

1. Let  $X = \mathbf{R}^2$ . Check if  $(X, d)$  is a metric space, where distance  $d : X \times X \rightarrow \mathbf{R}_+$  is defined by:

(i)  $d((x_1, x_2), (y_1, y_2)) = (x_1^2 + x_2^2)^{1/2} + (y_1^2 + y_2^2)^{1/2}$ ,

(ii)  $d((x_1, x_2), (y_1, y_2)) = |x_1 - y_1| + |x_2 - y_2|$ ,

(iii)  $d((x_1, x_2), (y_1, y_2)) = |x_2| + |y_2| + |x_1 - y_1|$ .

2. For each of the metric spaces defined in problem 1, each  $x \in X$  and  $\rho > 0$ , find  $B_\rho(x)$ .

3. Prove that in a metric space:

(i) A union of an arbitrarily large family of open sets is an open set. The same for the intersection of a finite family of open sets.

(ii) An intersection of an arbitrarily large family of closed sets is a closed set. The same for the union of an arbitrarily large family of closed sets.

4. Elaydi, Problem 4 pg 102.

5. Elaydi, Problem 17 pg 103.

6. Elaydi, Problem 18 pg 103.