

Homework 8

1. Let (Σ_2^+, d) be defined as in class.
 - (i) Prove that d is a metric on Σ_2^+ .
 - (ii) Prove that if $d(x, y) < 1/2^n$ then the sequences x and y must have equal $n + 1$ first elements.
 - (iii) Prove that if x and y agree on first $n + 1$ places, then $d(x, y) \leq 1/2^n$.
 - (iv) Is (ii) true with $d(x, y) \leq 1/2^n$?
2. Elaydi problem 4 pg 127.
3. Elaydi problem 7 pg 128.
4. Elaydi problem 12 pg 128.
5. Let \approx denote the relation of conjugacy. Prove that it is an equivalence relation, that is it has the following three properties:
 - (i) For every $f : (X, d) \rightarrow (X, d)$ we have $f \approx f$.
 - (ii) If $f \approx g$ then $g \approx f$.
 - (iii) If $f \approx g$ and $g \approx k$ then $f \approx k$.
6. We say that the function $f : (X, d) \rightarrow (Y, \rho)$ is Lipschitz continuous iff:
$$\exists L > 0 \forall x, y \in X \quad \rho(f(x), f(y)) \leq L \cdot d(x, y).$$
 - (i) Prove that every Lipschitz continuous function is continuous.
 - (ii) Is every continuous function necessarily Lipschitz continuous?
7. Assume that $f : (X, d) \rightarrow (X, d)$ and $g : (Y, \rho) \rightarrow (Y, \rho)$ are two functions which are conjugate through the homomorphism $h : X \rightarrow Y$ (that is, we have: $h \circ f = g \circ h$).
Prove that if h is Lipschitz continuous and g has sensitive dependence on initial conditions then also f has sensitive dependence on initial conditions.