Marta Lewicka, Math 5535, Winter 2005

Homework 9

1. Let Λ be the Cantor set of the logistic map F_μ, with $\mu > 2 + \sqrt{5}$. Prove that the following map $h : \Lambda \longrightarrow \Sigma_2^+$ is a homeomorphism:

$$\forall x \in \Lambda \quad h(x) = \{a_0, a_1, a_2, \ldots\} \qquad \text{where } a_n = \begin{cases} 0 & \text{if } F_\mu^{(n)}(x) \in I_0, \\ 1 & \text{if } F_\mu^{(n)}(x) \in I_1. \end{cases}$$

2. Show that similarity of matrices $\approx$ is an equivalence relation. That is, prove that for every $A, B, C \in M^{2\times 2}$ we have:

(i) $A \approx A$.
(ii) If $A \approx B$ then $B \approx A$.
(iii) If $A \approx B$ and $B \approx C$ then $A \approx C$.

3. For given numbers $\alpha, \beta \in \mathbf{R}$ such that $\alpha^2 + \beta^2 \neq 0$, let $\lambda = \sqrt{\alpha^2 + \beta^2}$ and let $\omega = \operatorname{arctg}(\beta/\alpha)$ (if $\alpha = 0$ we put $\omega = \pi/2$ for $\alpha > 0$ and $\omega = -\pi/2$ for $\alpha < 0$). Let $B \in M^{2\times 2}$ be given as:

$$B = \begin{bmatrix} \alpha & -\beta \\ \beta & \alpha \end{bmatrix}.$$

Prove that for every $n \geq 1$ there holds:

$$B^n = \lambda^n \cdot \begin{bmatrix} \cos n\omega & \sin n\omega \\ -\sin n\omega & \cos n\omega \end{bmatrix}.$$

4. Assume that $A \in M^{2\times 2}$ is given as below and that it does not have any (real) eigenvalue:

$$A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}.$$

Define: $\alpha = (a + d)/2$ and $\beta = \sqrt{4(ad - bc) - (a + d)^2}/2$. Prove that α is an eigenvalue of the following matrix $B \in M^{4\times 4}$:

$$B = \begin{bmatrix} a & b & \beta & 0 \\ c & d & 0 & \beta \\ -\beta & 0 & a & b \\ 0 & -\beta & c & d \end{bmatrix}.$$

5. Elaydi problems 1-5 pg 142.

6. Elaydi problems 7 and 9 page 142