

MIDTERM 1, Math 8801, Fall 2007

**Problem 1.** (10 points)

Let  $E$  be a normed space and let  $\{x_n\}_{n=1}^{\infty}$  be a sequence in  $E$  such that for every  $T \in E^*$ , the series  $\sum_n Tx_n$  converges. Consider the mapping:

$$\phi(T) = \sum_n Tx_n.$$

Is it always true that  $\phi \in E^{**}$ ?

**Problem 2.** (20 points)

Let  $E$  be a Banach space. Let  $E_0$  and  $E_1$  be two closed subspaces of  $E$  such that  $E_0 + E_1$  is closed. Prove that:

$$\exists C > 0 \quad \forall x \in E \quad \text{dist}(x, E_0 \cap E_1) \leq C(\text{dist}(x, E_0) + \text{dist}(x, E_1)).$$

**Problem 3.** (30 points)

Let  $E, F$  be two reflexive Banach spaces. Let  $A : \mathcal{D}(A) \rightarrow F$  be linear, closed and defined on a dense subspace  $\mathcal{D}(A) \subset E$ .

- (i) Prove that  $\mathcal{D}(A^*)$  is a dense subspace of  $F^*$ .
- (ii) Consider the adjoint to  $A^*$ , that is the operator:  $(A^*)^* : \mathcal{D}((A^*)^*) \rightarrow F^{**}$ . Prove that under the natural identification of  $E$  with  $E^{**}$  and  $F$  with  $F^{**}$ , we have:

$$(A^*)^* = A.$$