

MIDTERM 2, Math 8801, Fall 2007

Problem 1. (20 points)

State and prove the Lax-Milgram theorem.

Problem 2. (20 points)

Let E, F be two Banach spaces and let $T \in \mathcal{L}(E, F)$.

- (i) Prove that T^{**} is a linear extension of T .
- (ii) Prove that T is invertible iff T^* is invertible, and that in such case:

$$(T^{-1})^* = (T^*)^{-1}.$$

Problem 3. (20 points)

Prove that every compact self-adjoint operator T (on a Hilbert space) has an eigenvalue λ , satisfying: $|\lambda| = \|T\|$.