Section 7.1

#5 $\int te^{-3t}\,dt$

Let $u = t$ $\quad dv = e^{-3t}dt \Rightarrow \quad v = \frac{-1}{3}e^{-3t}$

$$\int te^{-3t}\,dt = \int u\,dv = uv - \int v\,du$$

$$= \frac{-t}{3}e^{-3t} + \int \frac{1}{3}e^{-3t}\,dt$$

$$\boxed{= \frac{-t}{3}e^{-3t} - \frac{1}{9}e^{-3t} + C}$$

#9 $\int \ln \sqrt[3]{x}\,dx$

Let $w = \sqrt[3]{x} \Rightarrow w^3 = x \Rightarrow dx = 3w^2dw$

$$\int \ln\sqrt[3]{x}\,dx = \int \ln w \cdot 3w^2\,dw$$

Let $u = \ln w$, $dv = 3w^2dw \Rightarrow v = w^3$

$$\int u\,dv = uv - \int v\,du = \ln w \cdot w^3 - \int w^3 \frac{1}{w}\,dw$$

$$= \ln w \cdot w^3 - \frac{1}{3}w^3 + C$$

$$\boxed{= \ln\sqrt[3]{x} \cdot x - \frac{1}{3}x + C}$$

#11 $\int \arctan 4t\,dt$

Let $w = 4t \Rightarrow dt = \frac{1}{4}dw$

$$= \int \arctan w \cdot \left(\frac{1}{4}dw\right) = \frac{1}{4}\int \arctan w\,dw$$

Let $u = \arctan w$; $dv = dw \Rightarrow v = w$

$$= \frac{1}{4}uv - \frac{1}{4}\int v\,du = \frac{1}{4}w \cdot \arctan w - \frac{1}{4}\int \frac{w}{1+w^2}\,dw$$

$$= \frac{1}{4}w \cdot \arctan w - \frac{1}{8}\ln|1+w^2| + C$$

$$\boxed{= t \cdot \arctan(4t) - \frac{1}{8}\ln|1+16t^2| + C}$$

\# 17 $I = \int e^{2\theta} \sin 3\theta \, d\theta$

$= \int \sin 3\theta \; d\frac{1}{2}e^{2\theta} \quad (u = \sin 3\theta \quad dv = d\frac{1}{2}e^{2\theta})$

$= \sin 3\theta \cdot \frac{1}{2}e^{2\theta} - \int \frac{1}{2}e^{2\theta} \cdot 3\cos 3\theta \, d\theta$

$= \frac{1}{2}e^{2\theta} \cdot \sin 3\theta - \int \frac{3}{2}\cos 3\theta \; d(\frac{1}{2}e^{2\theta})$

$= \frac{1}{2}e^{2\theta} \sin 3\theta - \frac{3}{4}\cos 3\theta \cdot e^{2\theta} + \int \frac{3}{4}e^{2\theta}(-3\sin 3\theta)\, d\theta + C$

$= \frac{1}{2}e^{2\theta}\sin 3\theta - \frac{3}{4}e^{2\theta}\cos 3\theta \underbrace{- \frac{9}{4}\int e^{2\theta}\sin 3\theta \, d\theta}_{= \frac{-9}{4}I} + C$

So $\frac{13}{4} I = \frac{1}{2}e^{2\theta}\sin 3\theta - \frac{3}{4}e^{2\theta} \cdot \cos 3\theta + C$

$\boxed{I = \frac{2}{13} e^{2\theta} \cdot \sin 3\theta - \frac{3}{13} e^{2\theta}\cos 3\theta + C \cdot \frac{4}{13}}$

\#21) $\int \frac{x e^{2x}}{(1+2x)^2} dx$

Let $u = x e^{2x}$; $dv = \frac{1}{(1+2x)^2} dx = d\frac{-1}{2(1+2x)} \Rightarrow v = \frac{-1}{2(1+2x)}$

$\Uparrow$

$(v = \int \frac{1}{(1+2x)^2} dx)$

So $\int u \, dv$

$= \frac{-xe^{2x}}{2(1+2x)} + \int \frac{1}{2(\cancel{1+2x})} (\cancel{1+2x}) e^{2x} dx$

$= \boxed{\frac{-xe^{2x}}{2(1+2x)} + \frac{1}{4}e^{2x} + C}$

#31 $\int_0^{1/2} \cos^{-1} x \, dx$

Let $u = \cos^{-1} x$, $dv = dx \Rightarrow v = x$ $\quad // \ \frac{-1}{\sqrt{1-x^2}} dx$

$$\Rightarrow \int_0^{1/2} \cos^{-1} x \, dx = x \cos^{-1} x \Big]_0^{1/2} - \int_0^{1/2} x \, d\cos^{-1} x$$

$$= \frac{1}{2} \cdot \frac{\pi}{3} + \int_0^{1/2} \frac{x \, dx}{\sqrt{1-x^2}} = \frac{\pi}{6} - \sqrt{1-x^2} \Big]_0^{1/2}$$

$$\boxed{= \frac{\pi}{6} - \frac{\sqrt{3}}{2} + 1}$$

#35 $\int_1^2 x^4 (\ln x)^2 \, dx$

Let $u = (\ln x)^2 \Rightarrow du = 2 \ln x \cdot \frac{1}{x}$

$dv = x^4 dx \Rightarrow v = \frac{1}{5} x^5$

$$\int_1^2 u \, dv = uv \Big]_1^2 - \int_1^2 v \, du$$

$$= \frac{1}{5} x^5 (\ln x)^2 \Big]_1^2 - \int_1^2 \frac{1}{5} x^5 \cdot 2 \ln x \cdot \frac{1}{x} dx$$

$$= \frac{32}{5} (\ln 2)^2 - \int_1^2 \frac{2}{5} x^4 \ln x \, dx$$

Let $s = \frac{2}{5} \ln x$ $\quad dr = x^4 dx = \frac{1}{5} dx^5$

$$= \frac{32}{5} (\ln 2)^2 - \frac{2}{25} \ln x \cdot x^5 \Big]_1^2 + \int_1^2 \frac{2}{25} x^5 \cdot \frac{1}{x} dx$$

$$= \frac{32}{5} (\ln 2)^2 - \frac{64}{25} \ln 2 + \frac{2}{125} x^5 \Big]_1^2$$

$$\boxed{= \frac{32}{5} (\ln 2)^2 - \frac{64}{25} \ln 2 + \frac{62}{125}}$$

P4 #41

$I = \int x \ln(1+x)dx$

Let $w = 1+x$ $\quad dw = dx \quad x = w-1$

$= \int (w-1) \ln w \, dw = \int w \cdot \ln w \, dw - \int \ln w \, dw$

For $\int w \cdot \ln w \, dw$

$= \int \ln w \, d\frac{1}{2}w^2 = \frac{1}{2}w^2 \cdot \ln w - \int \frac{1}{2}w^2 \, d\ln w$

$= \frac{1}{2}w^2 \cdot \ln w - \frac{1}{4}w^2 + C$

For $-\int \ln w \, dw$ $\quad$ Let $u = \ln w \quad dv = dw$

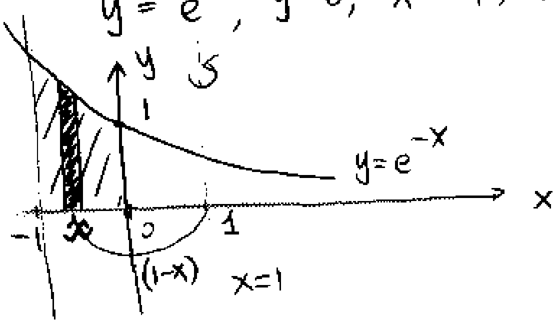
$= -\ln w \cdot w + \int w \cdot \frac{1}{w} dw = -\ln w \cdot w + w + C$

So, $I = \frac{1}{2}w^2 \cdot \ln w - \frac{1}{4}w^2 - w \ln w + w + C$

$\boxed{= \frac{1}{2}(1+x)^2 \ln(1+x) - \frac{1}{4}(1+x)^2 - (1+x)\ln(1+x) + (1+x) + C}$

63

$y = e^{-x}$, $y=0$, $x=-1$, $x=0$; about $x=1$



$ds = e^{-x}dx$

$r = (1-x)$

$d\text{volume} = 2\pi r \cdot ds$

$0 \geq x \geq -1$

Volume $= \int_{-1}^{0} 2\pi \cdot (1-x) \cdot e^{-x} dx$

Let $u = (1-x) \quad dv = e^{-x}dx$

$= 2\pi \left[(1-x)(-e^{-x}) + e^{-x} \right] \Big|_{-1}^{0}$

$= \boxed{2\pi e}$

P5

Section 7.2

#1 $\int \sin^2 x \cos^3 x \, dx$

$= \int \sin^2 x \, (\cos^2 x) \cos x \, dx = \int \sin^2 x \, (1-\sin^2 x) \, d\sin x$

let $w = \sin x$

$= \int w^2 (1-w^2) \, dw = \frac{1}{3}w^3 - \frac{1}{5}w^5 + C = \boxed{\frac{1}{3}\sin^3 x - \frac{1}{5}\sin^5 x + C}$

#7 $\int_0^{\pi/2} \cos^2\theta \, d\theta$ $\qquad \cos 2\theta = 2\cos^2\theta - 1$

$= \int_0^{\pi/2} \frac{1}{2}(1+\cos 2\theta)\, d\theta = \int_0^{\pi/2} \frac{1}{2} d\theta + \int_0^{\pi/2} \frac{1}{2}\cos 2\theta \, d\theta$

$= \frac{1}{2}\theta \Big]_0^{\pi/2} + \frac{1}{4}\sin 2\theta \Big]_0^{\pi/2} = \boxed{\frac{\pi}{4}}$

#13 $\int t \sin^2 t \, dt$ $\qquad \cos 2t = 1 - 2\sin^2 t$

$= \int t \left[\frac{1}{2}(1-\cos 2t)\right] dt = \int \frac{t}{2} dt - \int \frac{t}{2}\cos 2t \, dt$

$= \frac{1}{4}t^2 - \int \frac{t}{4} \, d\sin 2t = \frac{t^2}{4} - \frac{t}{4}\sin 2t + \int \sin 2t \cdot \frac{1}{4} dt$

$\boxed{= \frac{t^2}{4} - \frac{t}{4}\sin 2t - \frac{1}{8}\cos 2t + C}$

#21 $\int \tan x \sec^3 x \, dx$

$= \int \sec^2 x \cdot \tan x \sec x \, dx$ $\qquad d\sec x = \tan x \cdot \sec x \, dx$

$= \int \sec^2 x \, d\sec x$

$\boxed{= \frac{1}{3}\sec^3 x + C}$

#33 $\int x \sec x \cdot \tan x \, dx$

$= \int x \, d\sec x = x \cdot \sec x - \int \sec x \, dx$

$= \boxed{x \cdot \sec x - \ln|\sec x + \tan x| + C}$

#41 $\int \sin 8x \cdot \cos 5x \, dx$

$\sin A \cdot \cos B = \frac{1}{2}[\sin(A-B) + \sin(A+B)]$

$= \int \frac{1}{2}[\sin 3x + \sin 13x] \, dx$

$= \boxed{\frac{-1}{6} \cos 3x - \frac{1}{26} \cos 13x + C}$

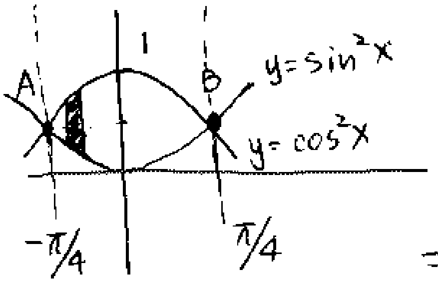
#47 $\int \frac{1-\tan^2 x}{\sec^2 x} \, dx = \int \frac{1 - \frac{\sin^2 x}{\cos^2 x}}{\cos^2 x} \, dx$

$= \int \frac{\frac{(\cos^2 x - \sin^2 x)}{\cos^2 x}}{\cos^2 x} \, dx = \int (\cos^2 x - \sin^2 x) \, dx$

$= \int \cos 2x \, dx = \boxed{\frac{1}{2} \sin 2x + C}$

#57 Find the area of the region bounded by:

$y_1 = \sin^2 x \quad y_2 = \cos^2 x \quad -\pi/4 \le x \le \pi/4$



$d\,\text{area} = dx(y_2 - y_1) = (\cos^2 x - \sin^2 x) \, dx$

$-\pi/4 \le x \le \pi/4$

$\Rightarrow \text{Area} = \int_{-\pi/4}^{\pi/4} (\cos^2 x - \sin^2 x) \, dx$

$= \frac{1}{2} \sin 2x \Big]_{-\pi/4}^{\pi/4} = \boxed{1}$

 Section 7.3

#5 $\int_{\sqrt{2}}^{2} \frac{dt}{t^3\sqrt{t^2-1}}$

let $t = \sec\theta$ $\quad t=\sqrt{2} \Rightarrow \cos\theta = \frac{\sqrt{2}}{2} \Rightarrow \theta = \pi/4$

$t = 2 \Rightarrow \cos\theta = 1/2 \Rightarrow \theta = \pi/3$

$I = \int_{\pi/4}^{\pi/3} \cos^2\theta\, d\theta = \left(\frac{1}{2}\theta + \frac{1}{4}\sin 2\theta\right)\Big]_{\pi/4}^{\pi/3} = \boxed{\frac{\pi}{24} + \frac{\sqrt{3}}{8} - \frac{1}{4}}$

#7 $\int_0^a \frac{dx}{(a^2+x^2)^{3/2}}$ $\quad a>0$

Let $x = a\tan\theta$ $\quad x=0 \Rightarrow \theta=0$; $x=a \Rightarrow \tan\theta = 1 \Rightarrow \theta = \pi/4$

$= \int_0^{\pi/4} \frac{a\sec^2\theta\, d\theta}{a^3(1+\tan^2\theta)^{3/2}} = \int_0^{\pi/4} a^{-2}\cos\theta\, d\theta = a^{-2}\sin\theta\Big]_0^{\pi/4}$

$= \boxed{\frac{\sqrt{2}}{2} a^{-2}}$

#11 $\int \sqrt{1-4x^2}\, dx$

$= \int \sqrt{1-(2x)^2}\, dx$

Let $2x = \sin\theta$ $\quad x = \frac{1}{2}\sin\theta$ $\quad dx = \frac{1}{2}\cos\theta\, d\theta$

$= \int \sqrt{1-\sin^2\theta}\ \frac{1}{2}\cos\theta\, d\theta = \int \frac{1}{2}\cos^2\theta\, d\theta$

$= \int \frac{1}{2}\cdot\frac{1}{2}(1+\cos 2\theta)\, d\theta = \boxed{\frac{1}{4}\theta + \frac{1}{8}\sin 2\theta + C}$

#15. $\int_0^a x^2\sqrt{a^2-x^2}\, dx$ $\quad$ let $x = a\sin\theta$

$0 \le x \le a \Rightarrow 0 \le \theta \le \pi/2$

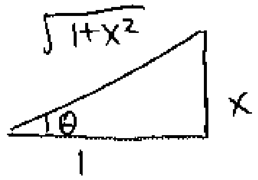
$= \int_0^{\pi/2} a^2\sin^2\theta\,(a\cos\theta)(a\cos\theta)\, d\theta$

$= a^4\int_0^{\pi/2} \sin^2\theta\cos^2\theta\, d\theta = a^4\int_0^{\pi/2} \frac{1}{4}\sin^2 2\theta\, d\theta$

$\sin 2\theta = 2\sin\theta\cos\theta$

$= \frac{a^4}{4}\int_0^{\pi/2} \frac{1}{2}(1-\cos 4\theta)\, d\theta = \frac{a^4}{8}\left(\theta - \frac{1}{4}\sin 4\theta\right)\Big]_0^{\pi/2} = \boxed{\frac{\pi}{16}a^4}$

#19 $\displaystyle\int \frac{\sqrt{1+x^2}}{x}\,dx$



Let $x = \tan\theta$, $-\frac{\pi}{2} < \theta < \frac{\pi}{2}$, $dx = \sec^2\theta\, d\theta$

$$I = \int \frac{\sec\theta \cdot \sec^2\theta}{\tan\theta}\,d\theta = \int \frac{\sec\theta\cdot(1+\tan^2\theta)}{\tan\theta}\,d\theta$$

$$= \int \frac{\sec\theta}{\tan\theta}\,d\theta + \int \tan\theta\cdot\sec\theta\,d\theta = \int \csc\theta\,d\theta + \int d\sec\theta$$

$$= \ln|\csc\theta - \cot\theta| + \sec\theta + C = \boxed{\ln\left|\frac{\sqrt{1+x^2}-1}{x}\right| + \sqrt{1+x^2} + C}$$

#21 $\displaystyle\int_0^{0.6} \frac{x^2}{\sqrt{9-25x^2}}\,dx$

$$= \int_0^{0.6} \frac{x^2}{\sqrt{9(1-\frac{25}{9}x^2)}}\,dx = \int_0^{0.6} \frac{x^2}{3\sqrt{1-(\frac{5}{3}x)^2}}\,dx$$

Let $w = \frac{5}{3}x \Rightarrow x = \frac{3}{5}w$ $\quad x=0 \Rightarrow w=0$; $x=0.6 \Rightarrow w=1$

$$= \int_0^1 \frac{\frac{9}{25}w^2\cdot\frac{3}{5}\,dw}{3\sqrt{1-w^2}} = \int_0^1 \frac{9}{125}\,\frac{w^2}{\sqrt{1-w^2}}\,dw$$

Let $w = \sin\theta$, $\quad w=0 \Rightarrow \theta=0 \quad w=1 \Rightarrow \theta = \pi/2$

$$= \int_0^{\pi/2} \frac{9}{125}\,\frac{\sin^2\theta}{\cos\theta}\cos\theta\,d\theta = \frac{9}{250}\left[\theta - \tfrac{1}{2}\sin 2\theta\right]\Big|_0^{\pi/2} = \boxed{\frac{9}{500}\pi}$$

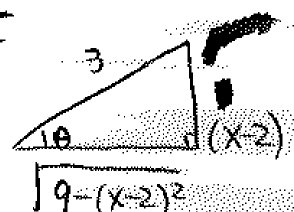
#23 $\displaystyle\int \sqrt{5+4x-x^2}\,dx$

Since $5+4x-x^2$ has coefficient $= -1$ for x^2, so we want to convert $5+4x-x^2$ into $a^2-(x-b)^2$

$$= \int \sqrt{5+4-4+4x-x^2}\,dx$$

$$= \int \sqrt{9-(x-2)^2}\,dx$$

$$= \int \sqrt{3^2-(x-2)^2}\,dx \qquad \text{Let } x-2 = 3\sin\theta,\ -\tfrac{\pi}{2} \le \theta \le \tfrac{\pi}{2}$$



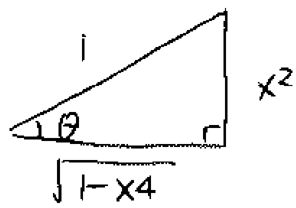
$$= \int 3\cos\theta\cdot 3\cos\theta\,d\theta = \frac{9}{2}\left[\theta + \tfrac{1}{2}\sin 2\theta\right] + C$$

$$= \boxed{\frac{9}{2}\left[\sin^{-1}\left(\frac{x-2}{3}\right)\right] + \frac{1}{2}(x-2)\sqrt{5+4x-x^2} + C}$$

p9

#29 $\int x\sqrt{1-x^4}\,dx$

$= \int x\sqrt{1-(x^2)^2}\,dx$



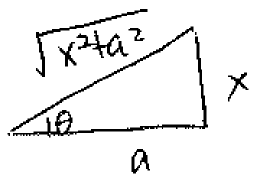
Let $w = x^2 \Rightarrow dw = 2x\,dx \Rightarrow x\,dx = \frac{1}{2}dw$

$= \int \sqrt{1-w^2}\ \frac{1}{2}dw \overset{w=\sin\theta}{=} \int \cos\theta\ \frac{1}{2}\cos\theta\,d\theta$

$= \int \frac{1}{4}[1+\cos 2\theta]\,d\theta = \frac{1}{4}\theta + \frac{1}{8}\sin 2\theta + C$

$= \frac{1}{4}\theta + \frac{1}{4}\sin\theta\cos\theta + C = \frac{1}{4}\sin^{-1}(x^2) + \frac{1}{4}x^2\sqrt{1-x4} + C$

#31 proof.



(a) $\int \frac{dx}{\sqrt{x^2+a^2}}$

Let $x = a\tan\theta$ $\quad dx = a\sec^2\theta\,d\theta$

$I = \int \frac{a\sec^2\theta\,d\theta}{a\sec\theta} = \int \sec\theta\,d\theta = \ln|\sec\theta + \tan\theta| + C$

$= \ln\left|\frac{\sqrt{x^2+a^2}}{a} + \frac{x}{a}\right| + C = \ln\left|\frac{1}{a}(\sqrt{x^2+a^2}+x)\right| + C$

$= \ln|\sqrt{x^2+a^2}+x| + \underbrace{\ln\frac{1}{a}}_{\text{constant}} + C = \ln|\sqrt{x^2+a^2}+x| + \tilde{C}$

(b) $\int \frac{dx}{\sqrt{x^2+a^2}}$

Let $x = a\sinh t$ $\quad dx = a\cosh t\,dt$ $\quad \sqrt{x^2+a^2} = a\cosh t$

So $I = \int \frac{a\cosh t}{a\cosh t}\,dt = t + C = \sinh^{-1}\left(\frac{x}{a}\right) + C$

Section 7.4

#7 $\int \frac{x^4}{x-1}\,dx = \int \left[(x^3+x^2+x+1) + \frac{1}{x-1}\right]dx$

$= \boxed{\frac{1}{4}x^4 + \frac{1}{3}x^3 + \frac{1}{2}x^2 + x + \ln|x-1| + C}$

$$\begin{array}{r|l} & x^3+x^2+x+1 \\ x-1 & x^4 \\ & \underline{x^4-x^3} \\ & x^3 \\ & \underline{x^3-x^2} \\ & x^2 \\ & \underline{x^2-x} \\ & x \\ & \underline{x-1} \\ & 1 \end{array}$$

#9 $\int \frac{5x+1}{(2x+1)(x-1)}\,dx$

$\frac{5x+1}{(2x+1)(x-1)} = \frac{A}{2x+1} + \frac{B}{x-1} = \frac{Ax-A+2Bx+B}{(2x+1)(x-1)}$

$\begin{cases} A+2B=5 & (1) \\ -A+B=1 & (2) \end{cases}$ $\quad (1)+(2) \Rightarrow 3B=6 \quad B=2 \quad A=1$

$I = \int \frac{1}{2x+1}\,dx + \int \frac{2}{x-1}\,dx = \boxed{\frac{1}{2}\ln|2x+1| + 2\ln|x-1| + C}$

13 $\int \frac{ax}{x^2-bx}\,dx$

$x^2-bx = x(x-b)$

$\Rightarrow \int \frac{ax}{x^2-bx}\,dx = \int \frac{a}{x-b}\,dx = \boxed{a\ln|x-b| + C}$

#17 $\int_1^2 \frac{4y^2-7y-12}{y(y+2)(y-3)}\,dy$

$\frac{4y^2-7y-12}{y(y+2)(y-3)} = \frac{A}{y} + \frac{B}{y+2} + \frac{C}{y-3} \Rightarrow \begin{cases} A+B+C=4 \\ -A-3B+2C=-7 \\ -6A=-12 \end{cases}$ $\quad A=2 \quad B=9/5 \quad C=1/5$

$= \int_1^2 \left[\frac{2}{y} + \frac{9}{5(y+2)} + \frac{1}{5(y-3)}\right]dy = 2\ln y + \frac{9}{5}\ln(y+2) + \frac{1}{5}\ln(y-3)\Big]_1^2$

P11

#19 $\displaystyle\int \frac{x^2}{(x-3)(x-2)^2}\,dx$

$$\frac{x^2}{(x-3)(x-2)^2} = \frac{A}{x-3} + \frac{B}{x-2} + \frac{C}{(x-2)^2} \Rightarrow \begin{cases} A+B=1 \\ -4A-5B+C=0 \\ 4A+6B-3C=1 \end{cases} \quad \begin{matrix} A=10 \\ B=-9 \\ C=-5 \end{matrix}$$

$$I = \int \frac{10}{x-3} - \frac{9}{x-2} - \frac{5}{(x-2)^2}\,dx = \boxed{10\ln|x-3| - 9\ln|x-2| + \frac{5}{x-2} + C}$$

#23 $\displaystyle\int \frac{10}{(x-1)(x^2+9)}\,dx$

$$\frac{10}{(x-1)(x^2+9)} = \frac{A}{x-1} + \frac{Bx+C}{(x^2+9)} = \frac{Ax^2+9A+Bx^2+Cx-Bx-C}{(x-1)(x^2+9)}$$

$$\begin{cases} A+B=0 \\ -B+C=0 \\ 9A-C=10 \end{cases} \Rightarrow \begin{cases} A=1 \\ B=-1 \\ C=-1 \end{cases}$$

$$I = \int \left(\frac{1}{x-1} - \frac{x+1}{x^2+9}\right)dx$$

$$= \int \frac{1}{x-1}\,dx - \int \frac{x\,dx}{x^2+9} - \int \frac{dx}{x^2+9}$$

$$= \boxed{\ln|x-1| - \frac{1}{2}\ln|x^2+9| - \frac{1}{3}\tan^{-1}\left(\frac{x}{3}\right) + C}$$

#27 $\displaystyle\int \frac{x^3+x^2+2x+1}{(x^2+1)(x^2+2)}\,dx$

$$\frac{x^3+x^2+2x+1}{(x^2+1)(x^2+2)} = \frac{Ax+B}{x^2+1} + \frac{Cx+D}{x^2+2}$$

$$\begin{cases} A+C=1 \\ B+D=1 \\ 2A+C=2 \\ 2B+D=1 \end{cases} \Rightarrow \begin{cases} A=1 \\ C=0 \\ B=0 \\ D=1 \end{cases}$$

$$I = \int \frac{x}{x^2+1} + \frac{1}{x^2+2}\,dx = \boxed{\frac{1}{2}\ln|x^2+1| + \frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{x}{\sqrt{2}}\right) + C}$$

#35 $\displaystyle\int \frac{dx}{x(x^2+4)^2}$

$$\frac{1}{x(x^2+4)^2} = \frac{A}{x} + \frac{Bx+C}{x^2+4} + \frac{Dx+E}{(x^2+4)^2}$$

$$1 = A(x^4+8x^2+16) + (Bx^2+Cx)(x^2+4) + x(Dx+E)$$

$$\begin{cases} A+B=0 \\ C=0 \\ 4C+E=0 \\ 16A=1 \end{cases} \Rightarrow \begin{matrix} A=\frac{1}{16} \\ B=\frac{-1}{16} \\ C=0 \\ D=\frac{-1}{4} \end{matrix}$$

$$I = \int \frac{1}{16x} + \frac{-x}{16(x^2+4)} - \frac{x}{4(x^2+4)^2}\,dx$$

$$= \frac{1}{16}\ln|x| - \frac{1}{32}\ln|x^2+4| + \frac{1}{8(x^2+4)} + C$$

Section 7.5

#1 $\int \cos x\,(1+\sin^2 x)\,dx$

Let $u = \sin x \quad du = \cos x\,dx$

$I = \int (1+u^2)\,du = u + \frac{1}{3}u^3 + C = \boxed{\frac{1}{3}\sin^3 x + \sin x + C}$

#5 $\int \frac{t}{t^4+2}\,dt = \int \frac{t}{(t^2)^2+2}\,dt$

Let $u = t^2 \quad du = 2t\,dt \quad t\,dt = \frac{1}{2}du$

$I = \int \frac{\frac{1}{2}du}{u^2+2} = \frac{1}{2}\frac{1}{\sqrt{2}}\tan^{-1}\left(\frac{u}{\sqrt{2}}\right) + C = \boxed{\frac{1}{2\sqrt{2}}\tan^{-1}\left(\frac{t^2}{\sqrt{2}}\right) + C}$

#11 $\int \frac{x-1}{x^2-4x+5}\,dx$

First for x^2-4x+5, $\Delta = (-4)^2 - 4\cdot1\cdot5 = 16-20 = -4 < 0$

so there is no further partial fractions for $\frac{x-1}{x^2-4x+5}$

$$\int \frac{x-1}{x^2-4x+5}\,dx = \int \frac{x\,dx}{x^2-4x+5} + \int \frac{-1\,dx}{x^2-4x+5}$$

$$= \int \frac{(x-2)\,dx}{x^2-4x+5} + \int \frac{2dx}{x^2-4x+5} + \int \frac{-1\,dx}{x^2-4x+5}$$

$$= \int \frac{(x-2)\,dx}{x^2-4x+5} + \int \frac{1\,dx}{x^2-4x+5}$$

For $\int \frac{x-2}{x^2-4x+5}\,dx$ let $u = x^2-4x+5 \Rightarrow du = (2x-4)\,dx$

$\Rightarrow (x-2)\,dx = \frac{1}{2}du$

$= \int \frac{\frac{1}{2}du}{u} = \frac{1}{2}\ln|u|$

For $\int \frac{1}{x^2-4x+5}\,dx = \int \frac{1}{(x-2)^2+1}\,dx = \tan^{-1}(x-2)$

$x^2-4x+4+1$

$I = \boxed{\frac{1}{2}\ln|x^2-4x+5| + \tan^{-1}(x-2) + C}$

#17 $\int_0^{\pi} t\cos^2 t\,dt$ $\qquad \cos 2t = 2\cos^2 t - 1$

$= \int_0^{\pi} t\,\frac{1}{2}(\cos 2t + 1)\,dt = \int_0^{\pi}\left[\frac{t}{2}\cos 2t + \frac{t}{2}\right]dt$

$= \int_0^{\pi} \frac{t}{2}\cos 2t\,dt + \int_0^{\pi}\frac{t}{2}\,dt = \boxed{\frac{\pi^2}{4}}$

1) $\int_0^{\pi}\frac{t}{2}\cos 2t\,dt$

Let $\frac{t}{2} = u$ $\quad \cos 2t\,dt = dv \Rightarrow dv = d\,\frac{1}{2}\sin 2t$

$I_1 = uv\Big]_0^{\pi} - \int_0^{\pi} v\,du = \frac{t}{2}\cdot\frac{\sin 2t}{2}\Big]_0^{\pi} - \int_0^{\pi}\frac{1}{2}\sin 2t\,\frac{1}{2}\,dt$

$= -\int_0^{\pi}\frac{1}{4}\sin 2t\,dt = \frac{1}{8}\cos 2t\Big]_0^{\pi} = 0$

2) $\int_0^{\pi}\frac{t}{2}\,dt = \frac{t^2}{4}\Big]_0^{\pi} = \frac{\pi^2}{4}$

#21 $\int \arctan\sqrt{x}\,dx$

Let $u = \sqrt{x}$ $\quad u^2 = x \Rightarrow dx = 2u\,du$

$I = \int \arctan u \cdot 2u\,du$ $\quad$ Let $w = \arctan u$ $\quad dv = 2u\,du = du^2$

$\int w\,dv = wv - \int v\,dw = u^2\arctan u - \int u^2\,d\arctan u$

$= u^2\arctan u - \int u^2\,\frac{1}{1+u^2}\,du = u^2\arctan u - \int \frac{u^2-1+1}{u^2+1}\,du$

$= u^2\arctan u - \int 1\,du + \int\frac{1}{1+u^2}\,du = \boxed{u^2\arctan u - u + \arctan u}$

#27

$\int\frac{dx}{1+e^x}$ $\quad$ Let $u = e^x$ $\quad du = e^x dx = u\,dx \Rightarrow dx = \frac{1}{u}du$

$= \int\frac{1}{1+u}\,\frac{1}{u}\,du = \int\frac{1}{u(u+1)}\,du$

$\frac{1}{u(u+1)} = \frac{A}{u} + \frac{B}{u+1} = \frac{Au + A + Bu}{u(u+1)}$ $\quad \begin{matrix} A+B=0 \\ A=1 \end{matrix} \Rightarrow \begin{matrix} A=1 \\ B=-1 \end{matrix}$

$\int\frac{1}{u(u+1)}\,du = \int\frac{du}{u} - \int\frac{du}{u+1} = \ln|u| - \ln|u+1| + C$

$= \ln|e^x| - \ln|e^x+1| + C = \boxed{x - \ln|1+e^x| + C}$

P14

#39 $$\int \frac{\sec\theta \tan\theta}{\sec^2\theta - \sec\theta}\, d\theta$$

$$= \int \frac{d\sec\theta}{\sec^2\theta - \sec\theta} \overset{u=\sec\theta}{=} \int \frac{du}{u^2-u} = \int \frac{du}{u(u-1)}$$

$$\frac{1}{u(u-1)} = \frac{A}{u} + \frac{B}{u-1} = \frac{Au - A + Bu}{u(u-1)} \qquad \begin{matrix} A+B=0 \\ -A = 1 \end{matrix} \Rightarrow \begin{matrix} A=-1 \\ B=1 \end{matrix}$$

$$I = \int \frac{-du}{u} + \int \frac{du}{u-1} = \ln|u-1| - \ln|u| + C$$
$$= \boxed{\ln|\sec\theta - 1| - \ln|\sec\theta| + C}$$

#43 $$\int \frac{\sqrt{x}}{1+x^3}\, dx \qquad \text{Let } u = \sqrt{x} \quad u^2 = x \quad dx = 2u\,du, \quad x^3 = u^6$$

$$= \int \frac{u}{1+u^6}\, 2u\,du = \int \frac{2u^2}{1+u^6}\, du = \int \frac{2u^2}{1+(u^3)^2}\, du$$

Let $w = u^3 \quad dw = 3u^2\,du \quad u^2 du = \frac{1}{3} dw$

$$= \int \frac{2\,\frac{1}{3}\,dw}{1+w^2} = \frac{2}{3}\int \frac{dw}{1+w^2} = \frac{2}{3}\tan^{-1}(w) + C = \frac{2}{3}\tan^{-1}(u^3) + C$$
$$= \boxed{\frac{2}{3}\tan^{-1}(x^{3/2}) + C}$$

#47 $$\int x^3(x-1)^{-4}\, dx \qquad \text{let } u = (x-1) \quad x = u+1 \quad dx = du$$

$$= \int (u+1)^3 u^{-4}\, du = \int (u^3 + 3u^2 + 3u + 1)\, u^{-4}\, du$$

$$= \int \left(\frac{1}{u} + \frac{3}{u^2} + \frac{3}{u^3} + \frac{1}{u^4}\right) du = \ln|u| - \frac{3}{u} - \frac{3}{2u^2} - \frac{1}{3u^3} + C$$

$$= \boxed{\ln|x-1| - 3(x-1)^{-1} - \frac{3}{2}(x-1)^{-2} - \frac{1}{3}(x-1)^{-3} + C}$$

Section 7.8

#1 Explain why each of the following is improper

(a) $\int_1^2 \frac{x}{x-1}\,dx$ $\frac{x}{x-1}$ is discontinuous at 1.

(b) $\int_0^{+\infty} \frac{1}{1+x^2}\,dx$. the interval is $[0,\infty)$

(c) $\int_{-\infty}^{+\infty} x^2 e^{-x^2}\,dx$: the interval is $(-\infty,+\infty)$

(d) $\int_0^{\pi/4} \cot x\,dx$; $\cot x = \frac{\cos x}{\sin x}$ is discontinuous at 0.

#5 $\int_3^{\infty} \frac{1}{(x-2)^{3/2}}\,dx$

$\frac{1}{(x-2)^{3/2}}$ is continuous on any $[3,a]$ $\forall a \geq 3$

so $\int_3^{+\infty} \frac{1}{(x-2)^{3/2}}\,dx = \lim_{a\to\infty} \int_3^a \frac{dx}{(x-2)^{3/2}} \overset{\lim_{a\to\infty}}{=} (-2)(x-2)^{\frac{-1}{2}}\Big]_3^a$

$= \lim_{a\to\infty} \frac{-2}{\sqrt{a-2}} + \frac{2}{\sqrt{3-2}} = \frac{2}{\sqrt{3-2}} = \boxed{2}$ convergent

#9 $\int_2^{+\infty} e^{-5p}\,dp$

e^{-5p} is continuous on any $[2,a]$. $\forall a \geq 2$

So $\int_2^{+\infty} e^{-5p}\,dp = \lim_{a\to\infty} \int_2^a e^{-5p}\,dp = \lim_{a\to\infty} \frac{-1}{5} e^{-5p}\Big]_2^a = \boxed{\frac{1}{5}e^{-10}}$

convergent

#13 $\int_{-\infty}^{\infty} x e^{-x^2}\,dx$

$= \int_{-\infty}^{0} x e^{-x^2}\,dx + \int_0^{+\infty} x e^{-x^2}\,dx$

$\int_{-\infty}^{0} x e^{-x^2}\,dx = \lim_{b\to-\infty} \int_b^0 x e^{-x^2}\,dx = \lim_{b\to-\infty} \frac{-1}{2} e^{-x^2}\Big]_b^0 = \frac{-1}{2}$

$\int_0^{+\infty} x e^{-x^2}\,dx = \lim_{a\to+\infty} \int_0^a x e^{-x^2}\,dx = \lim_{a\to\infty} \frac{-1}{2} e^{-x^2}\Big]_0^a = \frac{1}{2}$

$I = \frac{-1}{2} + \frac{1}{2} = \boxed{0}$ convergent

#21 $\int_1^{+\infty} \frac{\ln x}{x} dx$

$\frac{\ln x}{x}$ is continuous on any $[1, a]$ $\forall a \geq 1$

So $I = \lim_{a\to\infty} \int_1^a \frac{\ln x}{x} dx = \lim_{a\to\infty} \int_1^a \ln x \, d\ln x = \lim_{a\to\infty} \frac{1}{2}(\ln x)^2 \Big]_1^a = \boxed{+\infty}$

divergent

#25 $\int_e^{+\infty} \frac{1}{x(\ln x)^3} dx$

Note that $e > 1$ so $\frac{1}{x(\ln x)^3}$ is continuous on $[e, a]$ $\forall a \geq e$

so $I = \lim_{a\to\infty} \int_e^a \frac{d\ln x}{(\ln x)^3} = \lim_{a\to\infty} \frac{-1}{2}(\ln x)^{-2}\Big]_e^a$

$= \lim_{a\to\infty} \frac{-1}{2}(\ln a)^{-2} + \frac{1}{2} = \boxed{\frac{1}{2} \text{ convergent}}$

#35 $\int_0^3 \frac{dx}{x^2-6x+5} = \int_0^3 \frac{dx}{(x-1)(x-5)} = \int_0^1 \frac{dx}{(x-1)(x-5)} + \int_1^3 \frac{dx}{(x-1)(x-5)}$

For $\int_0^1 \frac{dx}{(x-1)(x-5)} = \lim_{t\to 1^-} \int_0^t \frac{dx}{(x-1)(x-5)}$

$\frac{1}{(x-1)(x-5)} = \frac{A}{x-1} + \frac{B}{x-5} = \frac{Ax-5A+Bx-B}{(x-1)(x-5)}$ $\quad A+B=0,\ -5A-B=1$ $\quad \begin{cases} A = \frac{-1}{4} \\ B = \frac{1}{4} \end{cases}$

$\lim_{t\to 1^-} \int_0^t \frac{-dx}{4(x-1)} + \frac{dx}{4(x-5)} = \lim_{t\to 1^-} \frac{-1}{4}\ln|x-1| + \frac{1}{4}\ln|x-5|\Big]_0^t = \infty$

so divergent.

#49 Use the comparison Thm to determine whether converg or diverg

$\int_0^{+\infty} \frac{x}{x^3+1} dx = \int_0^1 \frac{x}{x^3+1} dx + \int_1^{+\infty} \frac{x\,dx}{x^3+1} = I_1 + I_2$

I_1 is proper and I_2 is improper. So we focus on I_2

$I_2 = \int_1^{+\infty} \frac{x}{x^3+1} dx$ on $[1, +\infty)$ $\frac{x}{x^3+1} \leq \frac{x}{x^3} = \frac{1}{x^2}$

and $\int_1^{+\infty} \frac{1}{x^2} dx = \lim_{a\to\infty} \frac{-1}{x}\Big]_1^a = 1$ convergent

So $\int_1^{+\infty} \frac{x}{x^3+1} dx$ is convergent. $\boxed{\text{so } \int_0^{+\infty} \frac{x}{x^3+1} dx \text{ is convergent}}$

P17

#53 $\displaystyle\int_0^1 \frac{\sec^2 x}{x\sqrt{x}}\,dx$

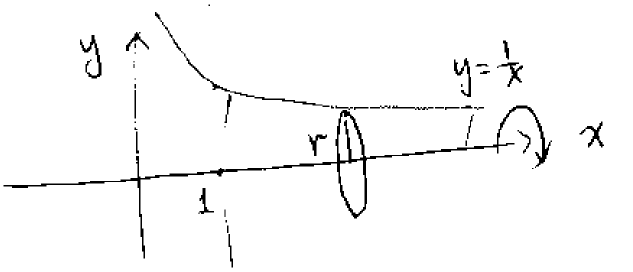
$\sec^2 x = \dfrac{1}{\cos^2 x}$ $\quad 0 \le \cos^2 x \le 1 \Rightarrow \sec^2 x \ge 1$

so $\dfrac{\sec^2 x}{x\sqrt{x}} \ge 1 \cdot \dfrac{1}{x\sqrt{x}}$

$\displaystyle\int_0^1 \frac{dx}{x\sqrt{x}} = \lim_{t\to 0^+} \int_t^1 x^{-\frac{3}{2}}\,dx = \lim_{t\to 0^+} (-2)\,x^{-\frac{1}{2}}\Big]_t^1 = \infty$

so $\displaystyle\int_0^1 \frac{dx}{x\sqrt{x}}$ is divergent and thus $\boxed{\displaystyle\int_0^1 \frac{\sec^2 x}{x\sqrt{x}}\,dx \text{ is diverg.}}$

#63 The region $R = \{(x,y) \mid x \ge 1 \quad 0 \le y \le 1/x\}$ has infinite area
show that by rotating R about the x-axis we obtain a solid with finite volume



$\displaystyle \text{Volume} = \int_1^{+\infty} d\text{volume}$

$\displaystyle = \int_1^{+\infty} \text{area(disk)}\,dx$

$\text{area(disk)} = \pi r^2 = \pi \dfrac{1}{x^2}$

so $\displaystyle \text{volume} = \int_1^{+\infty} \frac{\pi}{x^2}\,dx = \lim_{a\to\infty} \int_1^a \frac{\pi\,dx}{x^2}$

$\displaystyle = \lim_{a\to\infty} \frac{-\pi}{x}\Big]_1^a = \cancel{\frac{\pi}{a}}\ \boxed{\frac{+\pi}{1}}.$

$\boxed{\text{So the volume is } \pi,}$

P18

Section 8.1

#7 Find the exact length of the curve

$y = 1 + 6x^{3/2}$ $\quad y' = 6 \cdot \frac{3}{2} x^{\frac{1}{2}} = 9x^{\frac{1}{2}}$, $\quad 0 \le x \le 1$

$S = \int_0^1 \sqrt{1 + (\frac{dy}{dx})^2}\, dx = \int_0^1 \sqrt{1 + 81x}\, dx$

let $\quad u = \sqrt{1 + 81x}$ $\qquad x = 0 \quad u = 1 ; x = 1 \quad u = \sqrt{82}$

$u^2 = 1 + 81x \Rightarrow 2u\,du = 81\,dx \Rightarrow dx = \frac{2}{81} u\,du$

$S = \int_1^{\sqrt{82}} u \frac{2}{81} u\,du = \frac{2}{81 \cdot 3} u^3 \Big]_1^{\sqrt{82}} = \boxed{\frac{2}{243} [82^{\frac{3}{2}} - 1]}$

#9 $\quad y = \frac{x^3}{3} + \frac{1}{4x}$, $\quad 1 \le x \le 2$

$y' = x^2 + \frac{-1}{4x^2} \Rightarrow (y')^2 = x^4 - \frac{1}{2} + \frac{1}{16x^4}$

$S = \int_1^2 \sqrt{1 + (y')^2}\, dx = \int_1^2 \sqrt{1 + x^4 - \frac{1}{2} + \frac{1}{16x^4}}\, dx$

$= \int_1^2 \sqrt{x^4 + \frac{1}{2} + \frac{1}{16x^4}}\, dx = \int_1^2 \sqrt{(x^2 + \frac{1}{4x^2})^2}\, dx$

$= \int_1^2 (x^2 + \frac{1}{4x^2})\, dx = \boxed{\frac{x^3}{3} + \frac{-1}{4x} \Big]_1^2}$

#13. $\quad y = \ln(\sec x) \qquad 0 \le x \le \pi/4$

$y' = \frac{1}{\sec x} \cdot \sec x \tan x$ $\quad$ (chain rule)

$(y')^2 = \tan^2 x \Rightarrow \sqrt{1 + (y')^2} = \sqrt{1 + \tan^2 x} = \sqrt{\sec^2 x} = \sec x$

$S = \int_0^{\pi/4} \sqrt{1 + (y')^2}\, dx = \int_0^{\pi/4} \sec x\, dx = \boxed{\ln|\sec x + \tan x| \Big]_0^{\pi/4}}$

#15 $\quad y = \frac{1}{4}x^2 - \frac{1}{2}\ln x$, $\quad 1 \le x \le 2$

$y' = \frac{1}{2}x - \frac{1}{2x} \Rightarrow 1 + (y')^2 = 1 + \frac{x^2}{4} - \frac{1}{2} + \frac{1}{4x^2} = \frac{1}{2} + \frac{x^2}{4} + \frac{1}{4x^2}$

$= (\frac{x}{2} + \frac{1}{2x})^2$

$S = \int_1^2 \sqrt{1 + (y')^2}\, dx = \int_1^2 (\frac{x}{2} + \frac{1}{2x})\, dx = \boxed{\frac{x^2}{4} + \frac{1}{2}\ln x \Big]_1^2}$

#19. Find the length of the arc of the curve from point P to Q.

$y=\frac{1}{2}x^2 \quad P(-1,\frac{1}{2}) \quad Q(1,\frac{1}{2})$

Sol. First. $-1 \le x \le 1$; $y'=x$; $\sqrt{1+(y')^2} = \sqrt{1+x^2}$

so $S = \int_{-1}^{1} \sqrt{1+x^2}\, dx$ let $x=\tan\theta$ $\quad x=-1 \;\; \theta=\frac{-\pi}{4}$, $\quad x=1 \;\; \theta=\frac{\pi}{4}$

$= \underline{\int_{-\pi/4}^{\pi/4} \sec\theta \cdot \sec^2\theta\, d\theta} = \int_{-\pi/4}^{\pi/4} \sec\theta \cdot (1+\tan^2\theta)\, d\theta$

$= \int_{-\pi/4}^{\pi/4} \sec\theta\, d\theta + \int_{-\pi/4}^{\pi/4} \tan\theta \cdot d\sec\theta$

$= \ln|\sec\theta+\tan\theta|\Big]_{-\pi/4}^{\pi/4} + \tan\theta\cdot\sec\theta\Big]_{-\pi/4}^{\pi/4} - \underline{\int_{-\pi/4}^{\pi/4} \sec\theta\, \underbrace{d\tan\theta}_{\sec^2\theta d\theta}}$

Thus, $2\int_{-\pi/4}^{\pi/4} \sec^3\theta\, d\theta = \ln|\sec\theta+\tan\theta| + \tan\theta\cdot\sec\theta\Big]_{-\pi/4}^{\pi/4}$

$\Rightarrow \boxed{S = \int_{-\pi/4}^{\pi/4} \sec^3\theta\, d\theta = \frac{1}{2}\Big[\ln|\sec\theta+\tan\theta| + \tan\theta\cdot\sec\theta\Big]_{-\pi/4}^{+\pi/4}}$

#33. Find the arc length of the curve $y=2x^{3/2}$ starting with $P_0(1,2)$.

Sol. $y' = 2\cdot\frac{3}{2}x^{\frac{1}{2}} = 3x^{\frac{1}{2}}$ $\quad 1 \le x \le t$

let $u=\sqrt{1+9x}$

$u^2 = 9x+1$

$2u\,du = 9\,dx$

$x=1 \Rightarrow u=\sqrt{10}$

$x=t \Rightarrow u=\sqrt{1+9t}$

then $\int_1^t \sqrt{1+(y')^2}\, dx = \int_1^t \sqrt{1+9x}\, dx$

$= \int_{\sqrt{10}}^{\sqrt{1+9t}} u\cdot\frac{2u}{9}\,du = \frac{2}{27}u^3\Big]_{\sqrt{10}}^{\sqrt{1+9t}}$

$\boxed{= \frac{2}{27}\left[(1+9t)^{\frac{3}{2}} - 10\sqrt{10}\right]}$

P20

Section 8.2

#5 Find the surface by rotating the curve about the x-axis.

$y = x^3, \quad 0 \le x \le 2$

$y' = 3x^2 \qquad \text{Area} = \int_0^2 2\pi y \sqrt{1+\left(\frac{dy}{dx}\right)^2}\, dx = \int_0^2 2\pi x^3 \sqrt{1+9x^4}\, dx$

Let $1+9x^4 = u$ $\quad = \int_1^{145} \sqrt{u} \cdot \frac{2\pi}{36}\, du = \int_1^{145} \frac{\pi}{18} u^{\frac{1}{2}}\, du = \boxed{\left.\frac{\pi}{18}\cdot\frac{2}{3} u^{\frac{3}{2}}\right]_1^{145}}$

$36x^3 dx = du$

$x=0 \Rightarrow u=1; \quad x=2 \Rightarrow u=145$

#7 $y = \sqrt{1+4x} \qquad 1 \le x \le 5$

$y' = \frac{1}{2}\cdot(1+4x)^{-\frac{1}{2}}\cdot 4 = \frac{2}{\sqrt{1+4x}} \Rightarrow 1+(y')^2 = 1+\frac{4}{1+4x} = \frac{5+4x}{1+4x}$

$\text{Area} = \int_1^5 2\pi y \cdot \sqrt{1+(y')^2}\, dx = \int_1^5 2\pi \sqrt{1+4x}\cdot \frac{\sqrt{5+4x}}{\sqrt{1+4x}}\, dx = \int_1^5 \sqrt{5+4x}\, dx \cdot 2\pi$

let $5+4x = u \Rightarrow 4dx = du \qquad x=1 \Rightarrow u=9 \qquad x=5 \Rightarrow u=25$

$\text{Area} = \int_9^{25} \sqrt{u}\cdot \frac{2\pi}{4}\, du = \left.\frac{2\pi}{4}\left(u^{\frac{3}{2}}\cdot\frac{2}{3}\right)\right]_9^{25} = \frac{2\pi}{6}\left[125-27\right] = \boxed{\frac{98}{3}\pi}$

#9 ☆ $y = \sin \pi x \qquad 0 \le x \le 1$

$y' = \pi \cos \pi x \qquad 1+(y')^2 = 1+\pi^2\cos^2 \pi x$

$\text{Area} = \int_0^1 2\pi y \cdot \sqrt{1+(y')^2}\, dx = \int_0^1 2\pi \cdot \sin \pi x \sqrt{1+\pi^2\cos^2\pi x}\, dx$

$= \int_0^1 2\sqrt{1+\pi^2\cos^2 \pi x} \cdot \pi \sin \pi x\, dx$

Let $u = \pi \cos \pi x \Rightarrow du = d(\pi \cdot \cos \pi x) = -\pi^2 \sin \pi x\, dx \Rightarrow \pi \sin x\, dx = \frac{-1}{\pi} du$

$x=0 \quad u=\pi$

$x=1 \quad u=-\pi$

$\text{Area} = \int_\pi^{-\pi} 2\sqrt{1+u^2}\, \frac{-1}{\pi} du$ $\qquad \int_a^b f\, dx = -\int_b^a f dx$

$= \int_{-\pi}^{\pi} \frac{2}{\pi}\sqrt{1+u^2}\, du$

$= \boxed{\left.\frac{2}{\pi}\left[\frac{u}{2}\sqrt{1+u^2} + \frac{1}{2}\ln(u+\sqrt{1+u^2})\right]\right]_{-\pi}^{\pi}}$ (use formula directly or let $u = \tan\theta$)

P21

\# 13 obtain the surface by rotating the curve about the y-axis.

$$y = \sqrt[3]{x} = x^{\frac{1}{3}} \qquad 1 \le y \le 2$$

$$\Rightarrow x = y^3 \qquad \frac{dx}{dy} = 3y^2$$

let $u = 1 + 9y^4$

$y = 1 \quad u = 10$

$y = 2 \quad u = 145$

$du = 36y^3 dy$

$$\text{Area} = \int_1^2 2\pi x \sqrt{1 + \left(\frac{dx}{dy}\right)^2}\, dy = \int_1^2 2\pi y^3 \sqrt{1 + 9y^4}\, dy$$

$$= \int_{10}^{145} \sqrt{u} \cdot \frac{2\pi}{36}\, du = \frac{\pi}{18} \cdot \frac{2}{3} u^{\frac{3}{2}} \Big]_{10}^{145}$$

$$\boxed{= \frac{\pi}{27}\left[145\sqrt{145} - 10\sqrt{10}\right].}$$

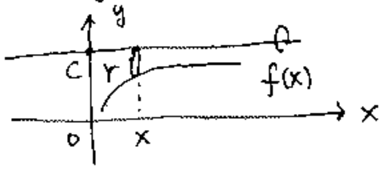
\# 15. $x = \sqrt{a^2 - y^2} \qquad 0 \le y \le a/2$

$$\frac{dx}{dy} = \frac{1}{2} \cdot (a^2 - y^2)^{-\frac{1}{2}} \cdot (-2y) \text{ (using chain rule) } = \frac{-y}{\sqrt{a^2 - y^2}}$$

$$\text{Area} = \int_0^{a/2} 2\pi \sqrt{a^2 - y^2} \cdot \sqrt{1 + \frac{y^2}{a^2 - y^2}}\, dy = \int_0^{a/2} 2\pi \sqrt{a^2 - y^2} \cdot \sqrt{\frac{a^2}{a^2 - y^2}}\, dy$$

$$= \int_0^{a/2} 2\pi a\, dy = 2\pi a y \Big]_0^{a/2} = \boxed{\pi a^2}$$

\#31 If the curve $y = f(x)$, $a \le x \le b$ is rotated about the horizontal line $y = c$, where $f(x) \le c$. Then find the formula for the area.



1) rotate about the horizontal line, so dx

2) the radius of rotation : $r = c - f(x)$

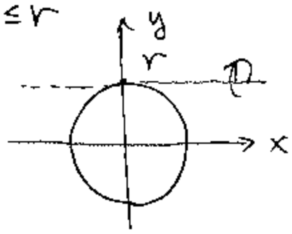
$$\text{so Area} = \int_a^b 2\pi r \sqrt{1 + \left(\frac{dy}{dx}\right)^2}\, dx$$

$$= \int_a^b 2\pi (c - f(x)) \sqrt{1 + (f'(x))^2}\, dx$$

\#33. Find the area of the surface by rotating $x^2 + y^2 = r^2$ about $y = r$.

1) rotating line : horizontal line $y = r$ so dx ; $-r \le x \le r$

~~then the area = 2 ∫_{-r} 2πy~~



2) $x^2 + y^2 = r^2 \Rightarrow y = \begin{cases} \sqrt{r^2 - x^2} & \text{upper circle} \\ -\sqrt{r^2 - x^2} & \text{lower circle} \end{cases}$

$$\text{Area}_{\text{upper}} = \int_{-r}^{r} 2\pi (r - \sqrt{r^2 - x^2}) \sqrt{1 + (y')^2}\, dx$$

(Different rotation radius!)

$$\text{Area}_{\text{lower}} = \int_{-r}^{r} 2\pi (r + \sqrt{r^2 - x^2}) \sqrt{1 + (y')^2}\, dx$$

$$\text{Area} = \text{Area}_{\text{upp}} + \text{Area}_{\text{low}} = \int_{-r}^{+r} 4\pi r \sqrt{1 + (y')^2}\, dx = \boxed{4\pi^2 r^2.}$$

section 9.1

#1 $y=\frac{2}{3}e^x+e^{-2x} \Rightarrow \frac{dy}{dx}=\frac{2}{3}e^x-2e^{-2x}$

Thus. LHS $=\frac{dy}{dx}+2y=\frac{2}{3}e^x-2e^{-2x}+\frac{4}{3}e^x+2e^{-2x}=\frac{6}{3}e^x=2e^x=$ RHS. #

#3 (a) $y=e^{rx}$ $y'=re^{rx}$ $y''=r^2e^{rx}$

$2y''+y'-y=0 \Rightarrow 2r^2+r-1=0 \Rightarrow (2r-1)(r+1)=0 \Rightarrow \boxed{r_1=\frac{1}{2}, r_2=-1}$

(b) $y=ae^{r_1x}+be^{r_2x} \Rightarrow y'=a(r_1e^{r_1x})+b(r_2e^{r_2x})$

$\Rightarrow y''=a(r_1^2e^{r_1x})+b(r_2^2e^{r_2x})$

$\Rightarrow 2y''+y'-y=a\underbrace{(2r_1^2e^{r_1x}+r_1e^{r_1x}-e^{r_1x})}_{=0}+b\underbrace{(2r_2^2e^{r_2x}+r_2e^{r_2x}-e^{r_2x})}_{=0}=0$

#5 (d): $y=\frac{-1}{2}x\cdot\cos x$

$y'=\frac{-1}{2}\cos x+\frac{1}{2}x\sin x$; $y''=\frac{1}{2}\sin x+\frac{1}{2}\sin x+\frac{1}{2}x\cos x$ (product rule)

so $y''+y=\sin x$. #

#7 (a) $y'=-y^2$

(1) $y\equiv 0$ is a solution.

(2) If y is not zero function, then $y'=-y^2<0 \Rightarrow y$ is strictly decreasing.

(b) $y=\frac{1}{x+c}$ $y'=\frac{-1}{(x+c)^2}=-y^2$. #

(c) For instance, $y\equiv 0$ is a solution that doesn't belong to family (b).

#9 (a) P is increasing $\Leftrightarrow \frac{dP}{dt}>0 \Leftrightarrow P(1-\frac{P}{4200})>0$, since $P>0$ is given (population cannot be negative!) so $1-\frac{P}{4200}>0 \Rightarrow 0\le P<4200$

(b) P is decreasing $\Leftrightarrow \frac{dP}{dt}<0 \Leftrightarrow P>4200$ (similar reason)

(c) P reaches equilibrium

$\Leftrightarrow \frac{dP}{dt}=0 \Leftrightarrow 1-\frac{P}{4200}=0 \Leftrightarrow P=4200$.

$\boxed{\text{So, the equilibrium solution is } 4200.}$

#11 $\frac{dy}{dt} = e^t (y-1)^2$

(1) when $y=1$ $\frac{dy}{dt} = 0$ $\Rightarrow$ slope of the curve at $y=1$ should be horizontal

$\Rightarrow$ exclude (b).

(2) $\frac{dy}{dt} = e^t (y-1)^2 \geq 0$ $\Rightarrow$ y cannot be decreasing, so exclude (a).

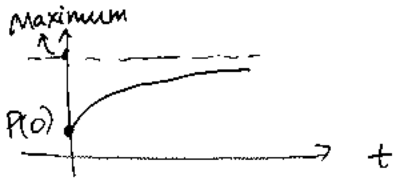
#15 (a) P increases most rapidly at the BEGINNING.

$\frac{dP}{dt}$ becomes smaller when t increase. (Guess :)

Since most people learn things fastest at the beginning.

But not true to all people, for example, I am always a slow learner :C

(c)



Section 9.2

#3 $y' = 2-y$

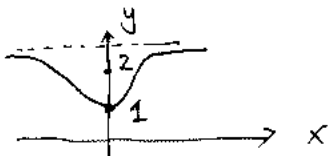
when $y=2$ $\frac{dy}{dx} = 0$ $\Rightarrow$ slope of the curve at $y=2$ should be horizontal.

$y=1$ $\frac{dy}{dx} = 1$ so (III).

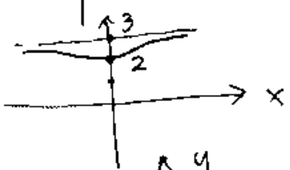
#5 $y' = x+y-1 \Rightarrow$ when $x=0, y=0$ $y' = 0+0-1 = -1$

so slope of the curve at $(0,0)$ should be -1, so (IV)

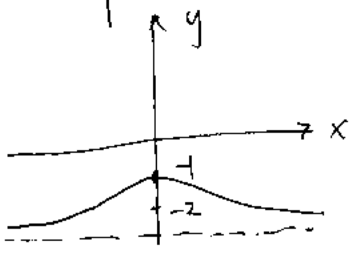
#7 (a) $y(0) = 1$



(b) $y(0) = 2$



(c) $y(0) = -1$



P24 #19 (a) Euler method:

$$\frac{y_{n+1}-y_n}{h} = f(y_n, x_n) \Rightarrow y_{n+1} = y_n + h\, f(y_n, x_n)$$

(1) $h=0.4$ $x_0=0$, $y_0=1$, need 1 step to reach 0.4 $\frac{dy}{dx} = y$

$x_0 \to x_1 = 0.4$

$y_0 \to y_1 = y_0 + h\cdot y_0 = 1 + 0.4\cdot 1 = 1.4$

(2) $h=0.2$ need 2 steps to reach 0.4

$x_0 \to \underset{0.2}{x_1} \to \underset{0.4}{x_2}$

$y_0 \to y_1 = y_0 + h\cdot y_0 \to y_2 = y_1 + h\cdot y_1$

$= 1.2$ $= 1.2 + 0.2\cdot 1.2 = 1.44$

(3) $h=0.1$ 4 steps are need

$x_0 \to x_1 \to x_2 \to x_3 \to x_4$

$0 \quad 0.1 \quad 0.2 \quad 0.3 \quad 0.4$

$y_0 \to y_1 = y_0 + h\cdot y_0 \to y_2 = y_1 + h\cdot y_1 \to y_3 = y_2 + h\cdot y_2 \to y_4 = y_3 + h\cdot y_3$

$y_1 = 1 + 0.1\cdot 1 = 1.1$

$y_2 = 1.1 + 0.1\cdot 1.1 = 1.21$

$y_3 = 1.21 + 0.1\cdot 1.21 = 1.331$

$y_4 = 1.331 + 0.1331 = \boxed{1.4641}$

(c) when step size is halved, the error is also approximately halved.

#21 $y' = y - 2x$ $y(1) = 0$ $h = 0.5$

~~$x_0 = 1 \to x_1 = 0.5 \to x_2 = 1 \to x_3 = 1.5$~~

~~$y_0 = 0 \to y_1 = y_0 + h(y_0 - 2x_0) \to y_2 = y_1 + h(y_1 - 2x_1)$~~

~~$= 0 + 0.5(0-2) = -1$ $= -1 + 0.5(-1-$~~

$x_0 = 1$ $y_0 = 0$

$x_1 = 1.5$ $y_1 = y_0 + h(y_0 - 2x_0) = 0 + 0.5(0-2) = -1$

$x_2 = 2$ $y_2 = y_1 + h(y_1 - 2x_1) = -1 + 0.5(-1-3) = -3$

$x_3 = 2.5$ $y_3 = y_2 + h(y_2 - 2x_2) = -3 + 0.5(-3-4) = -6.5$

$x_4 = 3$ $y_4 = y_3 + h(y_3 - 2x_3) = -6.5 + 0.5(-6.5-5) = -12.25$.

P25

Section 9.3

#3 $xy^2y' = x+1$

$y^2 \cdot y' = \frac{x+1}{x} = 1 + \frac{1}{x}$ when $x \neq 0$ #

$\Rightarrow \int y^2 dy = \int (1+\frac{1}{x})\, dx \Rightarrow \frac{1}{3}y^3 = x + \ln|x| + C$

$\Rightarrow \boxed{y = (3x + 3\ln|x| + C)^{1/3} \quad x \neq 0.}$ #

#7 $\frac{dy}{dt} = \frac{t}{ye^{y+t^2}} = \frac{t}{y \cdot e^y \cdot e^{t^2}}$

$\Rightarrow ye^y\, dy = \frac{t}{e^{t^2}}\, dt = t \cdot e^{-t^2} dt$

$\Rightarrow \int ye^y\, dy = \int te^{-t^2} dt$. LHS $= \int y\, de^y = ye^y - \int e^y dy = ye^y - e^y$

RHS $= \int te^{-t^2} dt = \int \frac{-1}{2} e^{-t^2} d(-t^2) = \frac{-1}{2} e^{-t^2} + C$

(or, let $u = -t^2$ $du = -2t\, dt$, so RHS $= \int \frac{-1}{2} e^u du = \frac{-1}{2} e^u$)

Thus, $\boxed{ye^y - e^y = \frac{-1}{2} e^{-t^2} + C}$

#13 $\frac{du}{dt} = \frac{2t + \sec^2 t}{2u}$ $u(0) = -5$

$\int 2u\, du = \int (2t + \sec^2 t)\, dt \Rightarrow u^2 = t^2 + \tan t + C$

$\Rightarrow u = \sqrt{t^2 + \tan t + C} \geq 0$ or $u = -\sqrt{t^2 + \tan t + C} \leq 0$

However $u(0) = -5 \Rightarrow C = 25$ and $\boxed{u = -\sqrt{t^2 + \tan t + 25}}$

#17 $y' \tan x = a + y$ $y(\pi/3) = a$ $0 < x < \pi/2$

$\int \frac{dy}{a+y} = \int \cot x\, dx = \int \frac{\cos x}{\sin x} dx = \int \frac{d \sin x}{\sin x}$

$\Rightarrow \ln|a+y| = \ln|\sin x| + C_1 \Rightarrow a + y = \underbrace{e^{C_1}}_{\text{constant}} \cdot \sin x = \underbrace{C_2}_{\text{constant}} \sin x$

Therefore $y(\pi/3) = a \Rightarrow a + y(\pi/3) = C_2 \sin\frac{\pi}{3} = \frac{\sqrt{3}}{2} C_2$

$C_2 = \frac{4a}{\sqrt{3}}$ and $\boxed{y = -a + \frac{4a}{\sqrt{3}} \sin x}$

#19. slope $= \frac{dy}{dx} = xy$; passing $(0,1) \Rightarrow y(0)=1$

$\frac{dy}{y} = x\,dx \Rightarrow \ln|y| = \frac{1}{2}x^2 + C_1$

$\Rightarrow y = C_2 e^{\frac{1}{2}x^2}$ $\quad y(0) = 1 = C_2 \cdot e^0 \Rightarrow C_2 = 1$

So $\boxed{y(x) = e^{\frac{1}{2}x^2}}$

#45

Let $y(t)$ = amout of salt in kg after t mins

$y(0) = 15$

$\frac{dy}{dt}$ = rate in − rate out.

rate in $= \underbrace{\frac{0\,kg}{10\,L}}_{\text{pure water}} \cdot \frac{10L}{min} = 0$; rate out $= \underbrace{\frac{y(t)}{1000L}}_{\text{percentage of salt in the brine}} \cdot \frac{10L}{min} = \frac{y(t)}{100}$ L/min

So $\begin{cases} \frac{dy}{dt} = 0 - \frac{y}{100} \\ y(0) = 15 \end{cases} \Rightarrow \begin{cases} y(t) = c\,e^{-\frac{t}{100}} \\ c = 15 \end{cases} \Rightarrow \boxed{y(t) = 15e^{-t/100}}$

#47 Let $y(t)$ = amount of alcohol in the vat after t mins.

$y(0) = 0.04 \cdot 500 = 20$ gal

$\frac{dy}{dt}$ = rate in − rate out

rate in $= 6\% \cdot 5$ gal/min $= 0.3$ gal/min ;

rate out $= \underbrace{\frac{y(t)}{500}}_{\text{percentage of beer}} \cdot 5$ gal/min $= \frac{y(t)}{100}$

so $\begin{cases} \frac{dy}{dt} = 0.3 - \frac{y}{100} = \frac{30-y}{100} \Rightarrow \frac{dy}{30-y} = \frac{dt}{100} \\ y(0) = 20 \end{cases}$

$\Rightarrow \boxed{y(t) = 30 - 10e^{-t/100}}$

section 9.4

\#1 (a) $\frac{dp}{dt} = 0.05p - 0.0005p^2 = 0.05p\left(1-\frac{p}{100}\right)$

Using P607 $\boxed{4}$ $\Rightarrow$ carrying capacity $M = 100$; $k \approx 0.05$

\#3 $\frac{dy}{dt} = ky\left(1-\frac{y}{M}\right) \Rightarrow y(t) = \frac{M}{1+Ce^{-kt}}$ $M = 8\times10^7$, $k = 0.71$

(a)

$y(0) = 2\times10^7$ kg $\Rightarrow C = 3 \Rightarrow y(t) = \frac{8\times10^7}{1+3\cdot e^{-0.71t}} \Rightarrow y(1) \cong 3.2\times10^7$ kg

(b) $4\times10^7 = y(t) = \frac{8\times10^7}{1+3\cdot e^{-0.71t}} \Rightarrow 1 = \frac{2}{1+3\cdot e^{-0.71t}} \Rightarrow 1+3e^{-0.71t} = 2$

$\Rightarrow e^{-0.71t} = \frac{1}{3} \Rightarrow t = 1.55$ years

---

\#5 $\frac{dy}{dt} = ky\left(1-\frac{y}{M}\right) \Rightarrow y(t) = \frac{10{,}000 = M}{1+Ce^{-kt}}$

$y(0) = 1000$, $M = 10{,}000$

$1000 = y(0) = \frac{10{,}000}{1+C\cdot e^{-0}} \Rightarrow C = 9 \Rightarrow y(t) = \frac{10{,}000}{1+9e^{-kt}}$

$2500 = y(1) = \frac{10{,}000}{1+9e^{-k\cdot 1}} \Rightarrow 1 = \frac{4}{1+9e^{-k}} \Rightarrow 1+9e^{-k} = 4$

$\Rightarrow k \approx -\ln\left(\frac{3}{9}\right) = -\ln(1/3) = \ln 3$

so, $y(4) = 9000$. (After another 3 years $\Rightarrow t = 1+3 = 4$)

\#7 The difference between birth and death rates is 20 million/year = 0.02 bill/year

so, according P606 and data in 1990

$k = \frac{1}{p}\frac{dp}{dt} \approx \frac{1}{p}\,\frac{\text{birth}-\text{death}}{\Delta t = 1\text{ year}} = \frac{1}{5.3}(0.02) = \frac{1}{265}$ (1 unit = 1 billion)

$M = 100$

so $\frac{dy}{dt} = kp\left(1-\frac{p}{M}\right) = \frac{1}{265}p\left(1-\frac{p}{100}\right) \Rightarrow y(t) = \frac{M}{1+Ce^{-kt}}$

$y(0) = 5.3 \Rightarrow C = 17.87$ so $y(10) = \frac{100}{1+17.87e^{-0.02\times10}} = 5.48$

$y(110) \approx 7.81$ $y(510) \approx 27.72$.

For $M = 50$, $\tilde{C} = \frac{M-y(0)}{y(0)} = \frac{50-5.3}{5.3}$, then do the same.

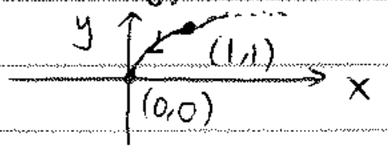
section 10.1

#3 $x=\cos^2 t \quad y=1-\sin t \quad 0\le t\le \pi/2$

Sol: $0\le t\le \pi/2 \Rightarrow 0\le \sin t\le 1 \Rightarrow$ y is from 1 to 0.

$\sin t = 1-y \Rightarrow 1=\cos^2 t+\sin^2 t = x+(1-y)^2$

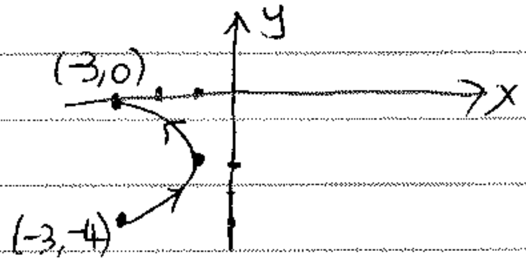
$x = 1-(1-y)^2 \quad 0\le y\le 1$ Init: $(1,1)$; end $(0,0)$



#7 $x=1-t^2 \quad y=t-2 \quad -2\le t\le 2$

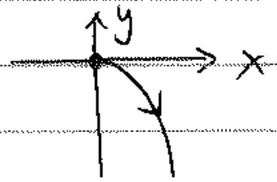
Sol $x=1-t^2 = 1-(y+2)^2$. Initial $(-3,-4)$ end $(-3,0)$

inbetween pt, $t=0 \Rightarrow (x,y)=(1,-2)$



#9 $x=\sqrt{t} \quad y=1-t$

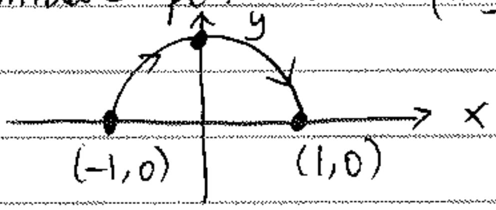
Sol $x=\sqrt{t} \Rightarrow x\ge 0 \quad y=1-t=1-x^2,\ x\ge 0$



#11 $x=\sin\frac{1}{2}\theta \quad y=\cos\frac{1}{2}\theta \quad -\pi\le\theta\le\pi$

$x^2+y^2=1$ initial $(-1,0)$ end $(1,0)$

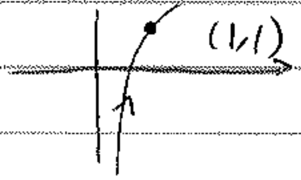
middle pt: $\theta=0 \ (x,y)=(0,1)$



P29

#15 $x = e^{2t}$ $y = t+1$

Sol: $2t = \ln x \Rightarrow t = \frac{1}{2}\ln x \Rightarrow y = 1 + \frac{1}{2}\ln x$, $x > 0$



#23 The curve is contained in the rectangle which is $1 \leq x \leq 4$; $2 \leq y \leq 3$.

Section 10.2

#1 $x = t\sin t$ $y = t^2 + t$

Sol: $dx/dt = \sin t + t\cos t$; $dy/dt = 2t+1$

So $\frac{dx}{dt} = 0 \Rightarrow \sin t + t\cos t = 0 \Rightarrow t = 0$

So, $\frac{dy}{dx} = \begin{cases} (2t+1)/(\sin t + t\cos t), & t \neq 0 \\ \text{vertical}, & t = 0 \end{cases}$

#3 $x = 1 + 4t - t^2$, $y = 2 - t^3$, $t = 1$

Sol. $t = 1$ $(x,y) = (4,1)$

slope of the line = tangent $= \left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{-3t^2}{4-2t}\right|_{t=1} = \frac{-3}{2}$

So the line expression. $(y-1) = \frac{-3}{2}(x-4)$

#5. $x = t\cos t$, $y = t\sin t$, $t = \pi$

Sol. $t = \pi$ $(x,y) = (-\pi, 0)$ $(\cos\pi = -1)$

$\frac{dy}{dx} = \left.\frac{\sin t + t\cos t}{\cos t - t\sin t}\right|_{t=\pi} = \frac{-\pi}{-1} = \pi$

So the line is. $y = \pi(x+\pi)$

#7 $x = 1 + \ln t$; $y = t^2 + 2$; $(1,3)$

Sol $(x,y) = (1,3) \Rightarrow t = 1$

(a) slope $= \left.\frac{dy}{dx}\right|_{t=1} = \left.\frac{2t}{+1/t}\right|_{t=1} = \left.2t^2\right|_{t=1} = 2$

so the line equation: $y - 3 = 2(x-1)$

(b) $x = 1 + \ln t \Rightarrow \ln t = x - 1 \Rightarrow t = e^{x-1}$

$y = t^2 + 2 = (e^{x-1})^2 + 2 = e^{2(x-1)} + 2$

solpe $= \left.\frac{dy}{dx}\right|_{x=1} = \left.2e^{2(x-1)}\right|_{x=1} = 2$ (chain rule : $u = x-1$)

#13 $x = e^t$ $y = te^{-t}$ Find $\frac{dy}{dx}$ & $\frac{d^2y}{dx^2}$; where concave upward (i.e $\frac{d^2y}{dx^2} > 0$)

Sol. $\frac{dy}{dx} = \frac{e^{-t} - te^{-t}}{e^t} = e^{-2t} - te^{-2t}$; $\frac{dx}{dt} = e^t$

$\frac{d^2y}{dx^2} = \frac{d}{dx}\left(\frac{dy}{dx}\right) = \frac{d}{dx}\left(e^{-2t} - te^{-2t}\right) = \frac{d(e^{-2t} - te^{-2t})}{dt}\frac{dt}{dx}$

$= (-2e^{-2t} - e^{-2t} + 2te^{-2t}) \cdot e^{-t} = -3e^{-3t} + 2te^{-3t}$

$= (2t-3)e^{-2t} > 0 \Rightarrow t > 3/2$ (since $e^{-2t} > 0$)

#17 Find out where tangent is horizontal or vertical

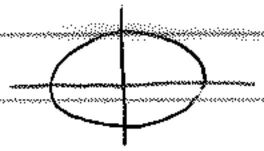
Sol: $x = t^3 - 3t$ $y = t^2 - 3$

$\frac{dx}{dt} = 3t^2 - 3$ $\frac{dy}{dt} = 2t$

Horizontal $\Rightarrow$ $\frac{dy}{dt} = 0$ AND $\frac{dx}{dt} \neq 0$ $\Rightarrow t = 0$

Vertical $\Rightarrow$ $\frac{dx}{dt} = 0$ AND $\frac{dy}{dt} \neq 0$ $\Rightarrow t = \pm 1$

P31

#31 Area of  $= 4 \cdot$ Area of 

with $0 \le x \le a \Leftrightarrow 0 \le \theta \le \pi/2$

 Area $= \left| \int_a^b y(x)\,dx \right| = \left| \int_\alpha^\beta y(t)\,dx(t) \right| = \left| \int_0^{\pi/2} -ab \sin^2\theta \, d\theta \right| = \frac{\pi ab}{4}$

so entire **area** $= \pi ab$

#33 . $x = 1 + e^t \quad y = t - t^2$

Sol. x-axis $\Rightarrow y = 0 \Rightarrow t - t^2 = 0,\ t_1 = 0,\ t_2 = 1$

Area $= \int_0^1 y(t)\, d\,x(t) = \int_0^1 (t - t^2)\, de^t = 3 - e$ (integration by parts)

#43 $x = t \sin t \quad y = t\cos t \quad 0 \le t \le 1$

Length $= \int_0^1 \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt = \int_0^1 \sqrt{(t\cos t + \sin t)^2 + (-t\sin t + \cos t)^2}$

$= \int_0^1 \sqrt{t^2 + 1}\, dt = \frac{\sqrt{2}}{2} + \frac{1}{2}\ln(1 + \sqrt{2})$

let $t = \tan\theta$, $0 \le \theta \le \pi/4$; then

do integration by parts for $\int \sec\theta \tan^2\theta\, d\theta$

#61 $x = t^3 \quad y = t^2 \quad 0 \le t \le 1$, rotate about x-axis

Area $= \int_0^1 2\pi y(t) \sqrt{\left(\frac{dx}{dt}\right)^2 + \left(\frac{dy}{dt}\right)^2}\, dt$

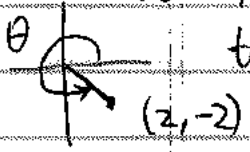
$= \int_0^1 2\pi t^2 \sqrt{9t^4 + 4t^2}\, dt = \int_0^1 2\pi t^2 \sqrt{9t^2 + 4}\; t\, dt$

let $u = t^2$ $= \int_0^1 \pi u \sqrt{9u + 4}\, du = \frac{2\pi}{1215}(247\sqrt{13} + 64)$

P32

section 10.3 :

#5 $r^2 = x^2 + y^2 = 4 + 4 = 8 \qquad r = 2\sqrt{2}$



$\tan\theta = \dfrac{y}{x} = -1 \ , \ 0 \le \theta < 2\pi \Rightarrow \theta = \dfrac{7}{4}\pi$

#17 $r = 2\cos\theta$

This is example 6 on P657

$(x-1)^2 + y^2 = 1$

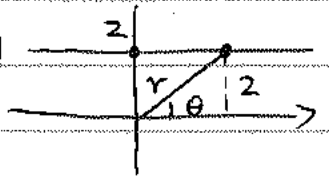
#19 $r^2 \cos 2\theta = 1$

$r^2 (2\cos^2\theta - 1) = 1 \qquad x = r\cos\theta \quad y = r\sin\theta \ , \ r^2 = x^2 + y^2$

$2r^2 \cos^2\theta - r^2 = 1 \qquad 2x^2 - (x^2 + y^2) = 1$

$x^2 - y^2 = 1$

#21



$r \cdot \sin\theta = 2 \Rightarrow r = \dfrac{2}{\sin\theta}$

So $r = 2\csc\theta$

#31 $r = 2(1 + \cos\theta)$

Sol: $\sqrt{x^2+y^2} = 2 + 2\dfrac{x}{\sqrt{x^2+y^2}}$

#39 $r = 1 - 2\sin\theta$

Sol: $\sqrt{x^2+y^2} = 1 - \dfrac{2y}{\sqrt{x^2+y^2}}$