Math and Climate Seminar IMA

Joint MCRN/IMA Math and Climate Seminar
Tuesdays 11:15 – 12:05
streaming video available at
`www.ima.umn.edu`

MCRN
www.mathclimate.org

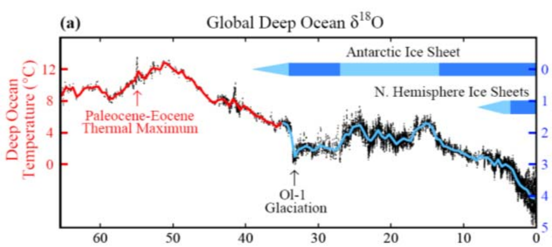
Celestial Influences on Glacial Cycles

Richard McGehee

Seminar on the Mathematics of Climate
IMA, MCRN, School of Mathematics
October 29, 2013

Glacial Cycles

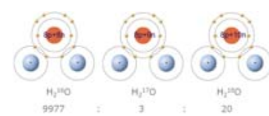
Temperatures in the Cenozoic Era



Hansen, et al, Target atmospheric CO2: Where should humanity aim? *Open Atmos. Sci. J.* 2 (2008)

Glacial Cycles

^{18}O as a Climate Proxy



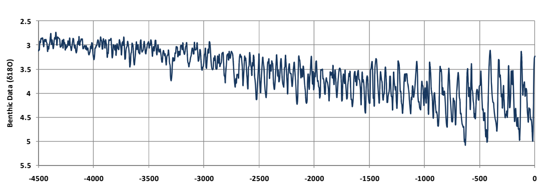
The isotope ^{16}O preferentially evaporates from the ocean and is sequestered in glaciers, leaving the heavier isotope ^{18}O more highly concentrated in the ocean. Thus oceanic concentration of the isotope ^{18}O is higher during glacial periods.

Foraminifera absorb more ^{18}O into their skeletons when the water temperature is lower and when more ^{18}O is in the water.

Thus higher concentrations of ^{18}O in foraminifera fossils indicate lower ocean temperatures and higher glacier volume.

Glacial Cycles

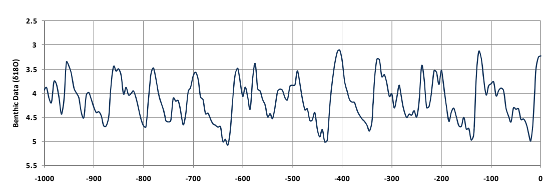
^{18}O in Foraminifera Fossils During the Past 4.5 Myr



Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

Glacial Cycles

^{18}O in Foraminifera Fossils During the Past 1.0 Myr

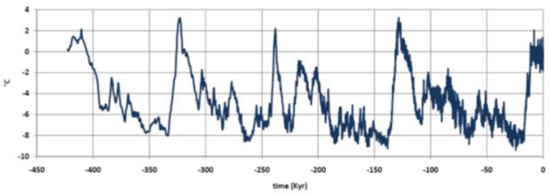


Lisiecki, L. E., and M. E. Raymo (2005), A Pliocene-Pleistocene stack of 57 globally distributed benthic d18O records, *Paleoceanography*,20, PA1003, doi:10.1029/2004PA001071.

Glacial Cycles

Recent (last 400 Kyr) Temperature Cycles

Vostok Ice Core Data



J.R. Petit, *et al* (1999) Climate and atmospheric history of the past 420,000 years from the Vostok ice core, Antarctica, *Nature* 399, 429-436.

Glacial Cycles

What Causes Glacial Cycles?

Widely Accepted Hypothesis

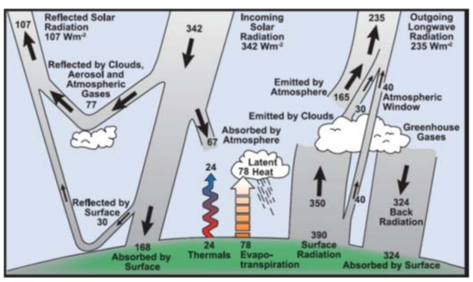
The glacial cycles are driven by the variations in the Earth's orbit (Milankovitch Cycles), causing a variation in incoming solar radiation (insolation).

This hypothesis is widely accepted, but also widely regarded as insufficient to explain the observations.

The additional hypothesis is that there are feedback mechanisms and/or triggering mechanisms that amplify the Milankovitch cycles. What these feedbacks are and how they work are not fully understood.

Glacial Cycles

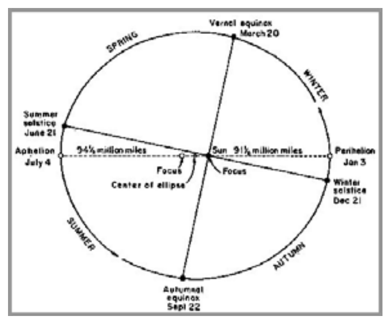
Heat Balance



Historical Overview of Climate Change Science, IPCC AR4, p.96
http://ipcc-wg1.ucar.edu/wg1/Report/AR4WG1_Print_CH01.pdf

Glacial Cycles

Eccentricity



John Imbrie & Katherine Palmer Imbrie, *Ice Ages: Solving the Mystery,* Harvard Univ. Press, 1979.

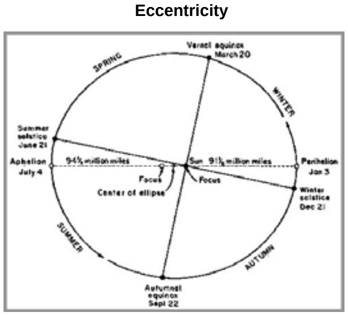
Glacial Cycles

Eccentricity

Perihelion: 91.5×10^6 mi
Aphelion: 94.5×10^6 mi

Semimajor axis: 93×10^6 mi

Eccentricity: 1.5/93 = 0.016



Glacial Cycles

Eccentricity

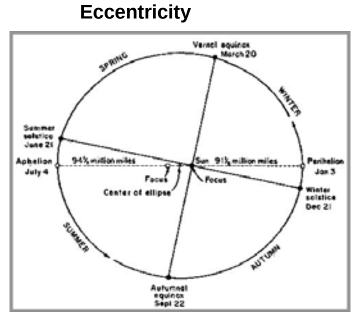
Perihelion: 91.5
Aphelion: 94.5

Change in radius:
3/93 = 3.2%

Change in insolation:
6.4%

Six percent less insolation in the southern winter than the northern winter.

6.4% of 342 W/m^2 =
22 W/m^2

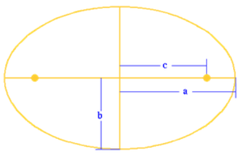


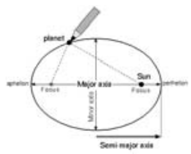
Glacial Cycles

Earth's Orbit

Kepler's First Law: The orbit of every planet is an ellipse with the Sun at one of the two foci.

Johannes Kepler (1571-1630)





Eccentricity = c/a

Glacial Cycles

Global Annual Average Insolation

Solar output: $K \approx 4\times10^{26}$ Watts

Solar intensity at distance r from the sun:

$$Q(t)=\frac{K}{4\pi r(t)^2}\quad \mathrm{Wm^{-2}}$$

Cross section of Earth: πr_E^2 $\mathrm{m^2}$

Global solar input: $\frac{K r_E^2}{4 r(t)^2}$ W

Total annual solar input (P = one year (in seconds)):

$$\int_0^P \frac{K r_E^2}{4 r(t)^2}dt=\frac{K r_E^2}{4}\int_0^P \frac{dt}{r(t)^2}\quad \text{Joules}$$

Glacial Cycles

Global Annual Average Insolation

Specific angular momentum (angular momentum per unit mass):

$$\Omega = r^2\dot{\theta}\quad \mathrm{m^2s^{-1}}$$

Total annual solar input:

$$\frac{K r_E^2}{4}\int_0^P \frac{dt}{r(t)^2}=\frac{K r_E^2}{4}\int_0^P \frac{\dot{\theta}dt}{\Omega}=\frac{K r_E^2}{4\Omega}\int_0^{2\pi} d\theta=\frac{\pi K r_E^2}{2\Omega}\quad \text{Joules}$$

Mean annual solar input:

$$\frac{\pi K r_E^2}{2P\Omega}\quad \text{Watts}$$

Mean annual solar intensity on the Earth's surface:

$$\frac{\pi K r_E^2}{2P\Omega}\cdot\frac{1}{4\pi r_E^2}=\frac{K}{8P\Omega}\quad \mathrm{Wm^{-2}}$$

Glacial Cycles

Global Annual Average Insolation

Kepler's Third Law:

$P \sim a^{-3/2}$ $\quad a$ = semimajor axis

Derived from Kepler:

$1-e^2 \sim a\Omega^2$ $\quad e$ = eccentricity

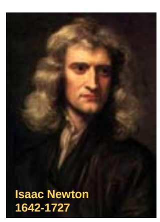
Mean annual solar intensity:

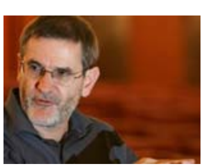
$$\frac{K}{8P\Omega}=\frac{\hat{K}a^{3/2}a^{1/2}}{\sqrt{1-e^2}}=\frac{\hat{K}a^2}{\sqrt{1-e^2}}\quad \mathrm{Wm^{-2}}$$

Glacial Cycles

Planetary Motion

$$m_i\frac{d^2x_i}{dt^2}=\sum_{\substack{j=1\\ j\neq i}}^{n}\frac{Gm_im_j\left(x_j-x_i\right)}{\left|x_j-x_i\right|^3}$$





Jacques Laskar (1955-)

The orbits of all the planets can be computed (both forward and backward in time) for billions of years.

Glacial Cycles

Global Annual Average Insolation

Laskar: $\frac{\hat{K}a^2}{\sqrt{1-e^2}}$

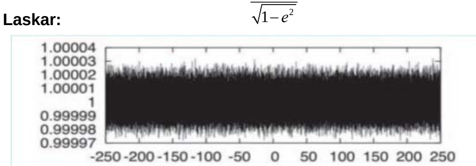


Fig. 11. Variation of the semi-major axis of the Earth–Moon barycenter (in AU) from −250 to +250 Myr.

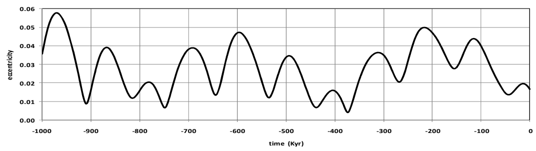
Semi major axis does not change much:

.005% corresponding to .01% change in global average insolation

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

Glacial Cycles

Eccentricity



Note periods of about 100 kyr and 400 kyr.

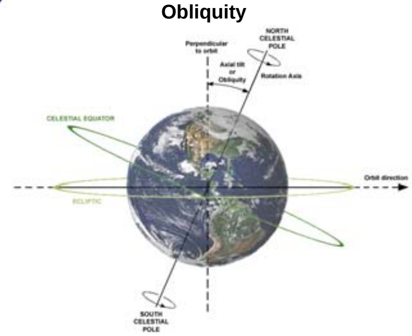
The effect due to eccentricity is more significant, but not that much:

As e varies between 0 and 0.06, $(1-e^2)^{-1/2}$ varies between 1 and 1.0018, or about 0.2%. (Twenty times the effect due to a.)

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

Glacial Cycles

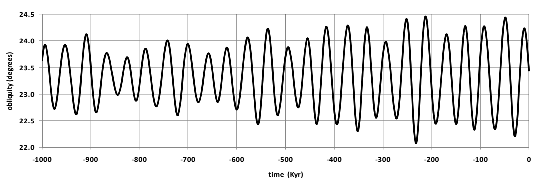
Obliquity



http://upload.wikimedia.org/wikipedia/commons/6/61/AxialTiltObliquity.png

Glacial Cycles

Obliquity

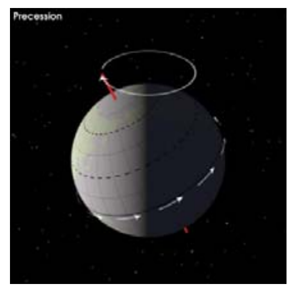


Note period of about 41 Kyr.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

Glacial Cycles

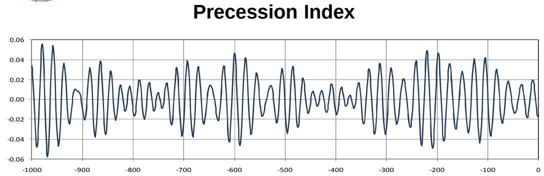
Precession



http://earthobservatory.nasa.gov/Library/Giants/Milankovitch/milankovitch_2.html

Glacial Cycles

Precession Index

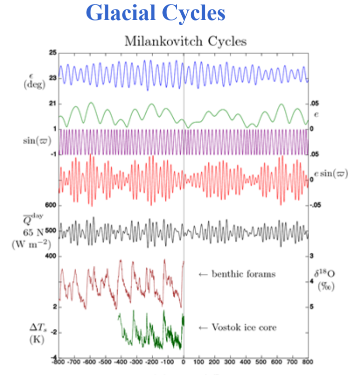


index = $e \sin\rho$, where e = eccentricity and ρ = precession angle (measured from spring equinox)

Note period of about 23 Kyr.

J. Laskar, et al (2004) A long-term numerical solution for the insolation quantities of the Earth, *Astronomy & Astrophysics* **428**, 261–285.

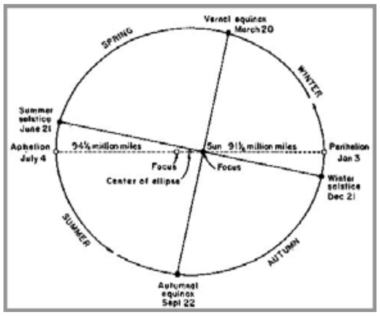
Glacial Cycles



http://en.wikipedia.org/wiki/Milankovitch_cycles

Glacial Cycles

Eccentricity



John Imbrie & Katherine Palmer Imbrie, *Ice Ages: Solving the Mystery,* Harvard Univ. Press, 1979.

Glacial Cycles

Daily Average Insolation at Summer Solstice at 65° N

Insolation at a point on the Earth's surface

$$I(\beta,\rho,r,\theta,\varphi,\gamma)=\frac{K}{4\pi r^2}\left[-\cos\varphi\left(\cos\beta\cos(\theta-\rho)\cos\gamma+\sin(\theta-\rho)\sin\gamma\right)-\sin\varphi\sin\beta\cos(\theta-\rho)\right]^+$$

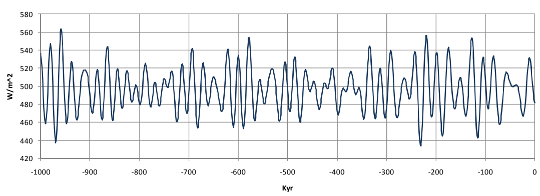
(φ,γ) = (latitude, longitude)
(r,θ) = position of Earth in orbital plane
β = obliquity angle
ρ = precession angle

Daily average insolation at latitude φ at summer solstice

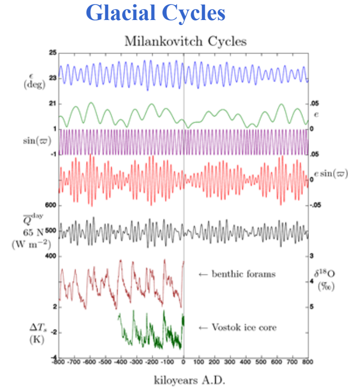
$$\bar{I}(e,\beta,\rho',\varphi)=Q\frac{(1-e\sin\rho')^2}{(1-e^2)^2}\frac{1}{2\pi}\int_0^{2\pi}\left[\cos\varphi\cos\beta\cos\gamma+\sin\varphi\sin\beta\right]^+ d\gamma$$

Glacial Cycles

Daily Average Insolation at Summer Solstice at 65° N



Glacial Cycles



http://en.wikipedia.org/wiki/Milankovitch_cycles

Glacial Cycles

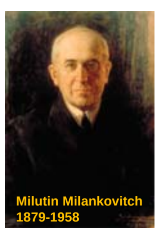
Who was Milankovitch?

Milutin Milankovitch was a Serbian mathematician and professor at the University of Belgrade.

In 1920 he published his seminal work on the relation between insolation and the Earth's orbital parameters.

In 1941 he published a book explaining his entire theory.

His work was not fully accepted until 1976.



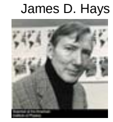
Glacial Cycles

What happened in 1976?

Hays, Imbrie, and Shackleton, "Variations in the Earth's Orbit: Pacemaker of the Ice Ages," *Science* **194**, 10 December 1976.

"It is concluded that changes in the earth's orbital geometry are the fundamental cause of the succession of Quaternary ice ages."

James D. Hays

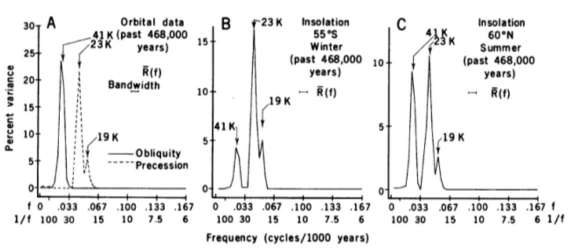


John Imbrie

Nicholas Shackleton

Glacial Cycles

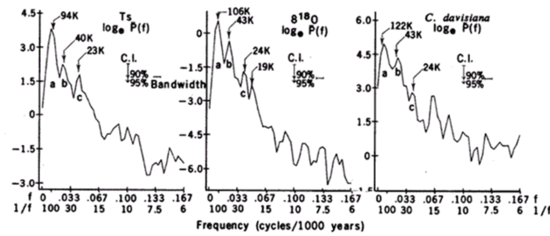
Solar Forcing (Hays, et al)



Hays, *et al*, *Science* **194** (1976), p. 1125

Glacial Cycles

Climate Response, Hays, et al



Three different temperature proxies from sea sediment data.

Hays, *et al*, *Science* **194** (1976), p. 1125

Glacial Cycles

Hays, et al, Summary

1) Three indices of global climate have been monitored in the record of the past 450,000 years in Southern Hemisphere ocean-floor sediments.

2) ... climatic variance of these records is concentrated in three discrete spectral peaks at periods of 23,000, 42,000, and approximately 100,000 years. These peaks correspond to the dominant periods of the earth's solar orbit, and contain respectively about 10, 25, and 50 percent of the climatic variance.

Hays, *et al*, *Science* **194** (1976), p. 1125

Glacial Cycles

Hays, et al, Summary

3) The 42,000-year climatic component has the same period as variations in the obliquity of the earth's axis and retains a constant phase relationship with it.

4) The 23,000-year portion of the variance displays the same periods (about 23,000 and 19,000 years) as the quasiperiodic precession index.

5) The dominant, 100,000-year climatic component has an average period close to, and is in phase with, orbital eccentricity. Unlike the correlations between climate and the higher-frequency orbital variations (which can be explained on the assumption that the climate system responds linearly to orbital forcing), an explanation of the correlation between climate and eccentricity probably requires an assumption of nonlinearity.

Hays, *et al*, *Science* **194** (1976), p. 1125

Glacial Cycles

Hays, et al, Summary

6) It is concluded that changes in the earth's orbital geometry are the fundamental cause of the succession of Quaternary ice ages.

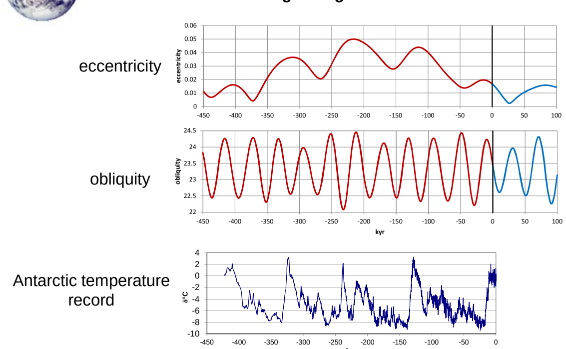
7) A model of future climate based on the observed orbital-climate relationships, but ignoring anthropogenic effects, predicts that the long-term trend over the next seven thousand years is toward extensive Northern Hemisphere glaciation*.

*Quoted by George Will, Washington Post, February 5, 2009

Hays, *et al*, *Science* **194** (1976), p. 1125

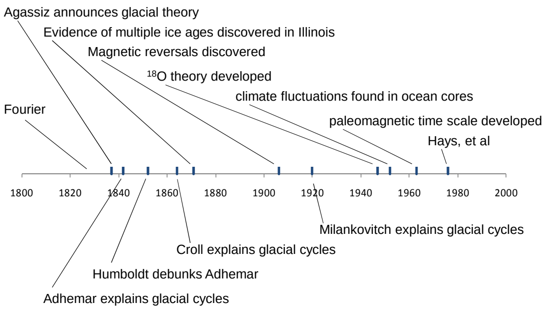
Glacial Cycles

The Coming Ice Age



Glacial Cycles

History of Discovery



Glacial Cycles

Church of Saint Sulpice, Paris

Glacial Cycles

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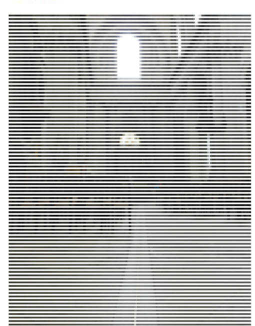
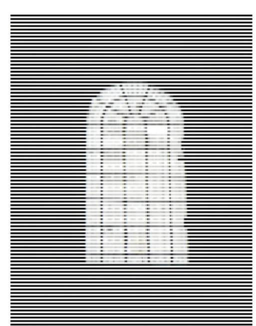
Glacial Cycles

Church of Saint Sulpice, Paris





Glacial Cycles

Next Week

Glacial Cycles: Theory Since 1976.