


Math and Climate Seminar IMA




Mathematics and Climate Research Network

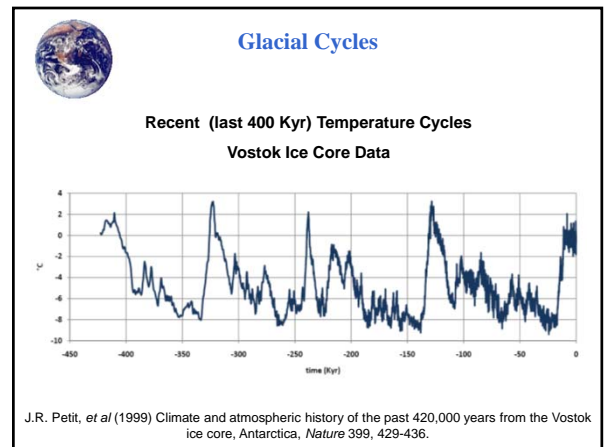
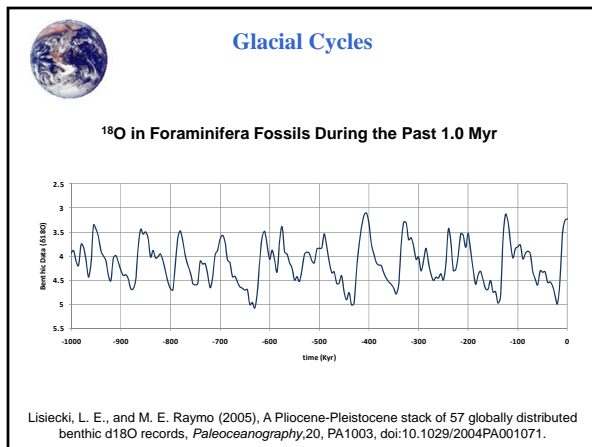
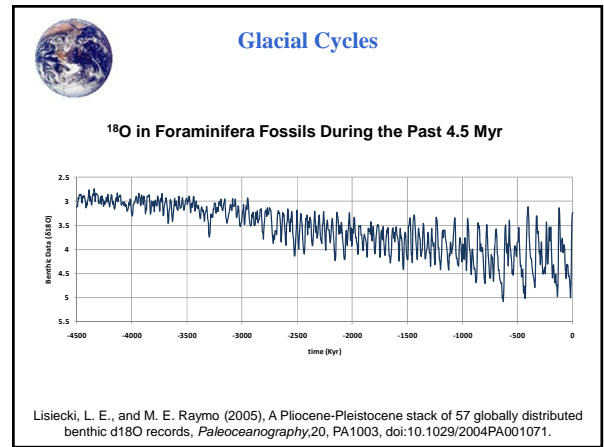
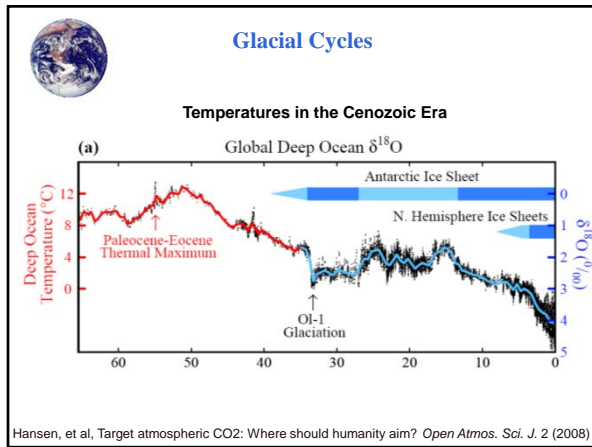
**Joint MCRN/IMA Math and Climate Seminar**  
 Tuesdays 11:15 – 12:05  
 streaming video available at  
 www.ima.umn.edu

MCRN www.mathclimate.org

**Recent Developments in the Theory of Glacial Cycles**  
 Richard McGehee



Seminar on the Mathematics of Climate  
 IMA, MCRN, School of Mathematics  
 November 5, 2013



**Glacial Cycles**

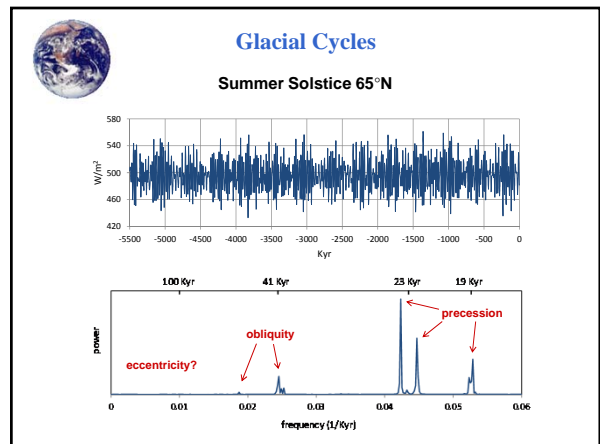
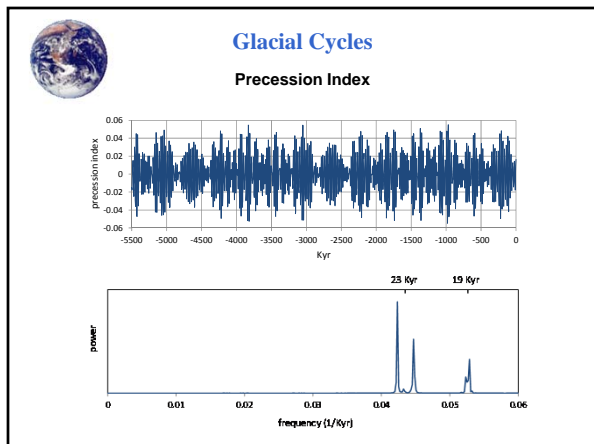
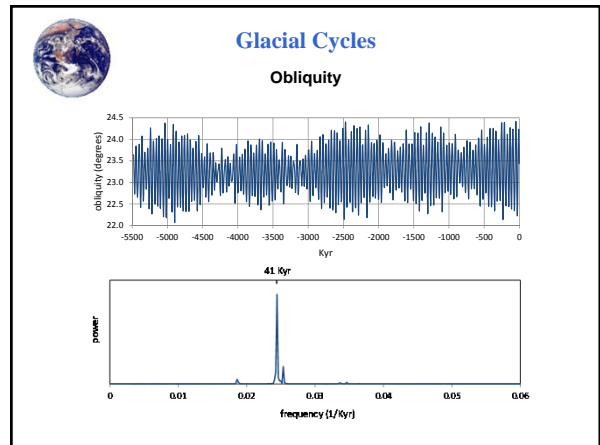
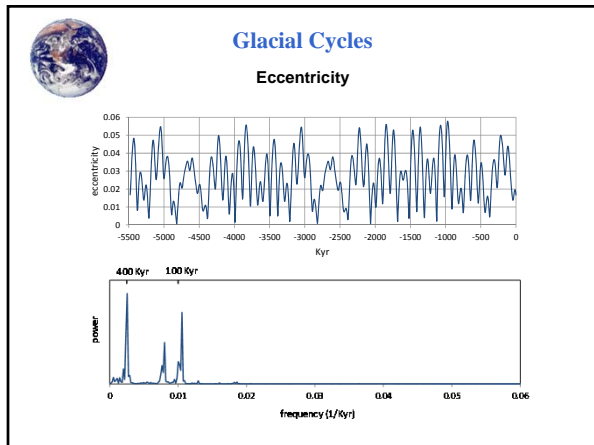
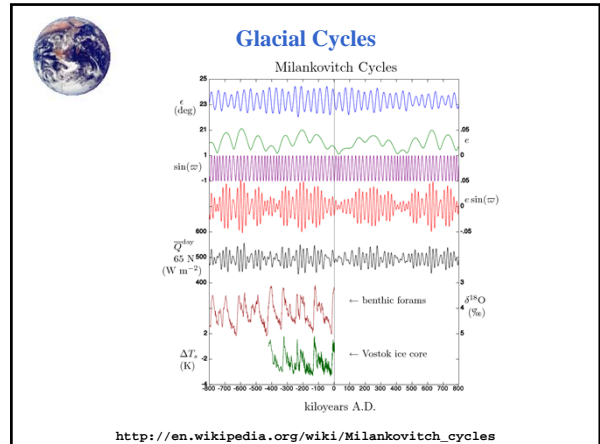
**What Causes Glacial Cycles?**

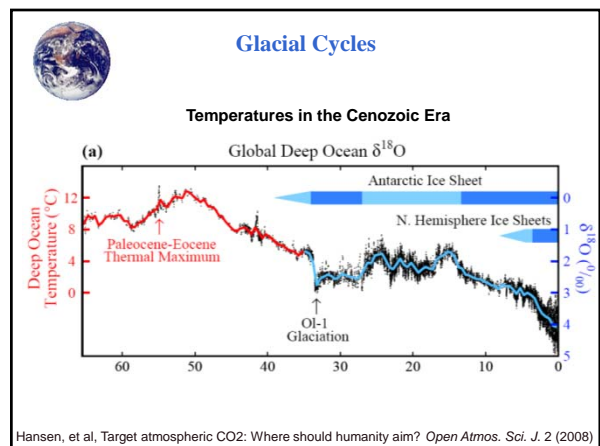
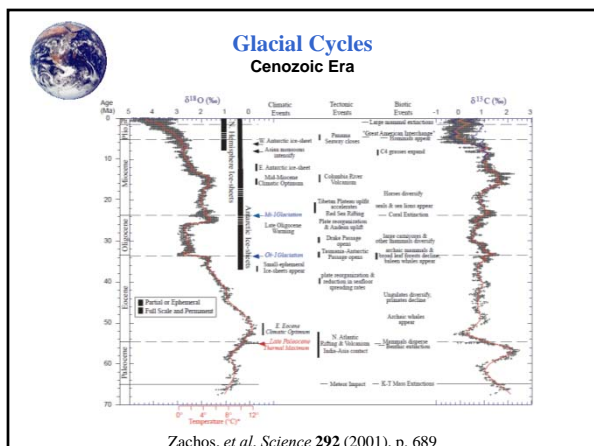
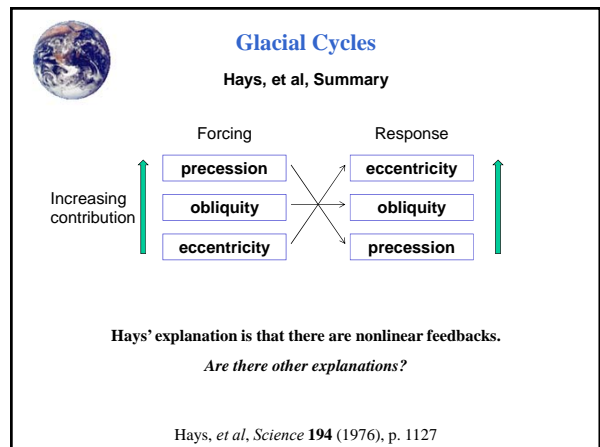
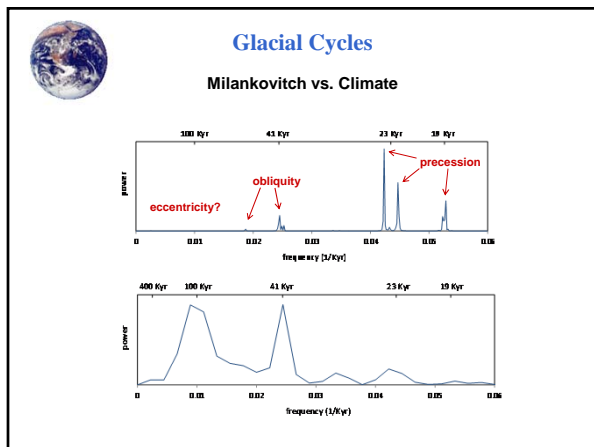
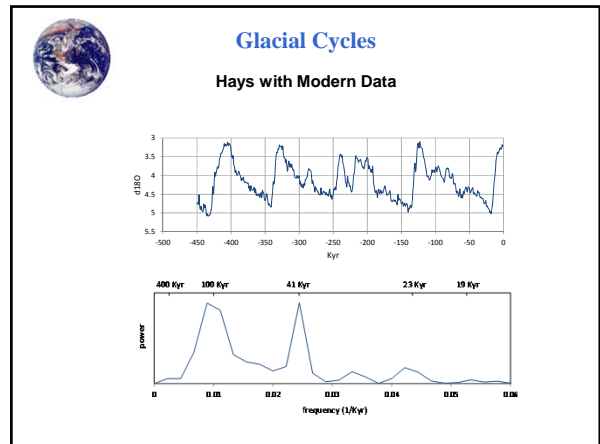
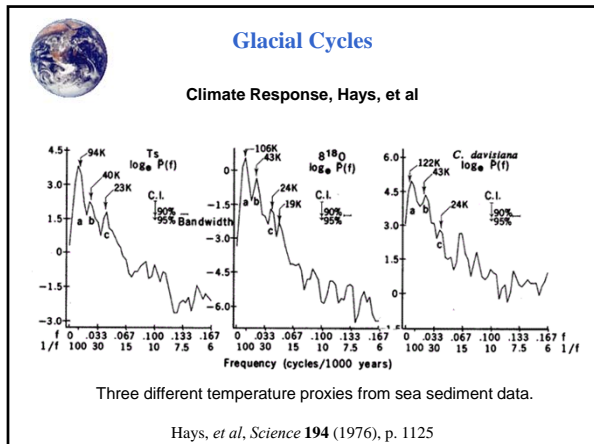
**Widely Accepted Hypothesis**

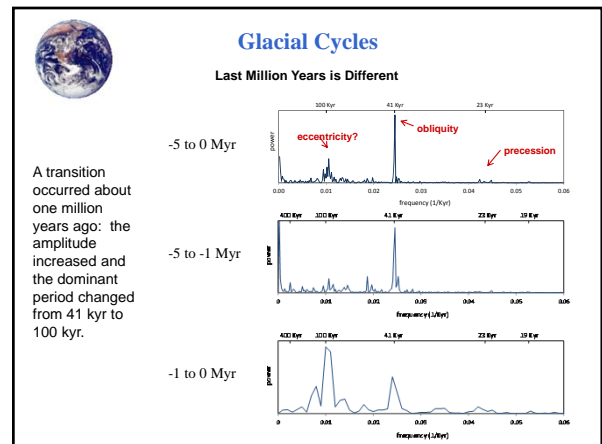
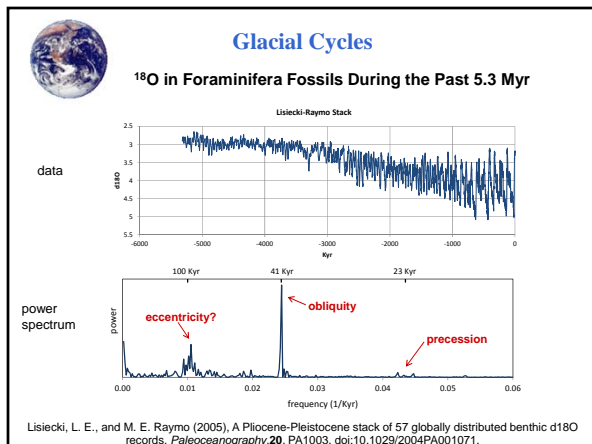
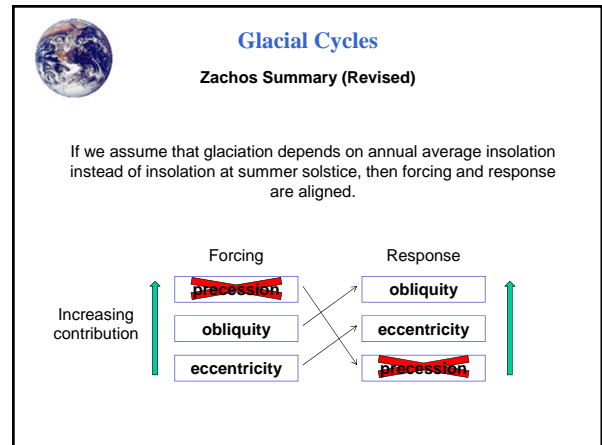
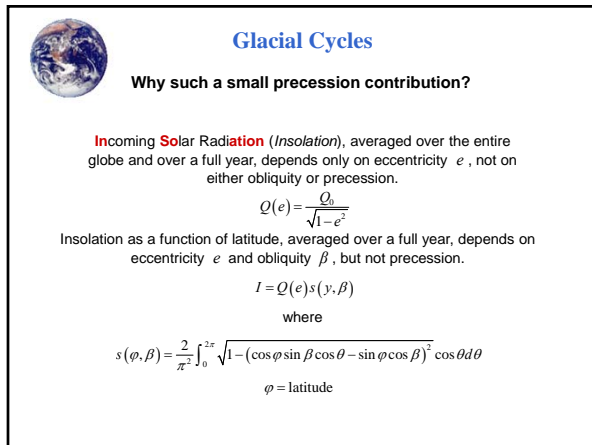
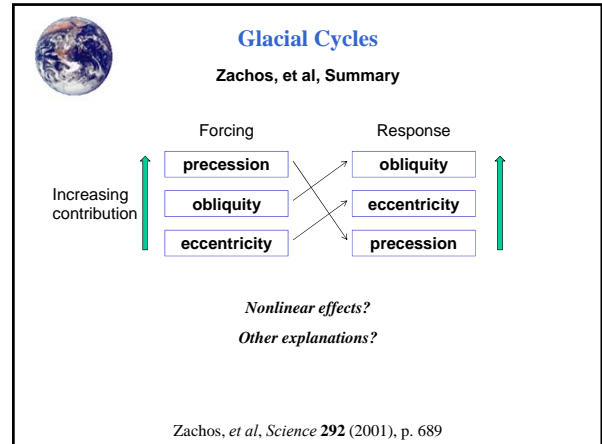
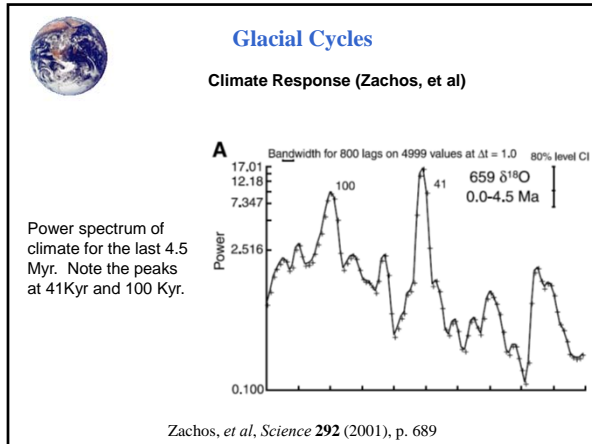
The glacial cycles are driven by the variations in the Earth's orbit (Milankovitch Cycles), causing a variation in incoming solar radiation (insolation).

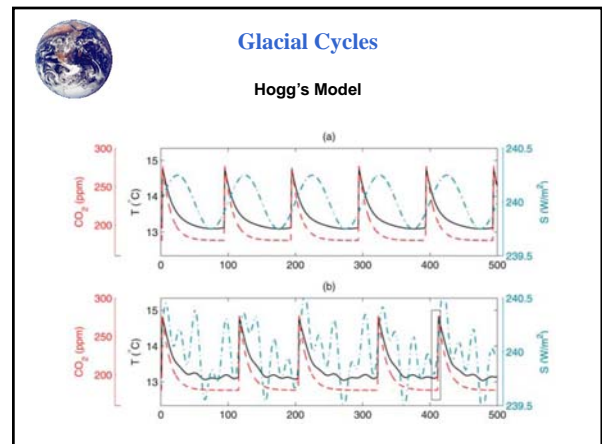
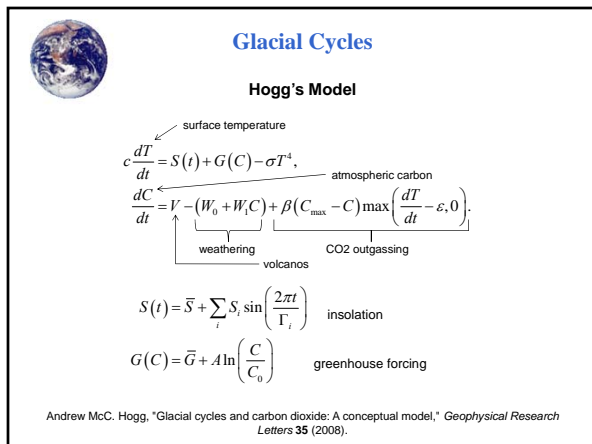
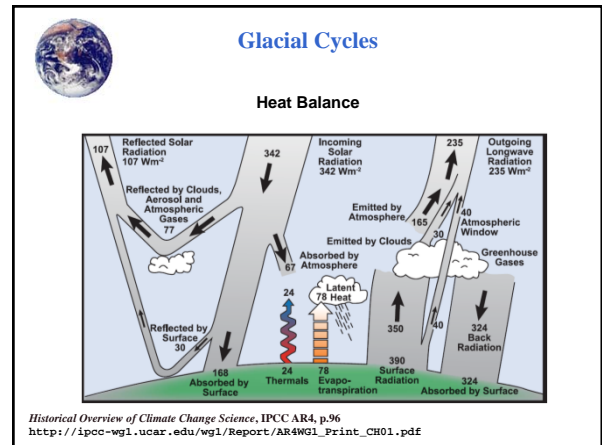
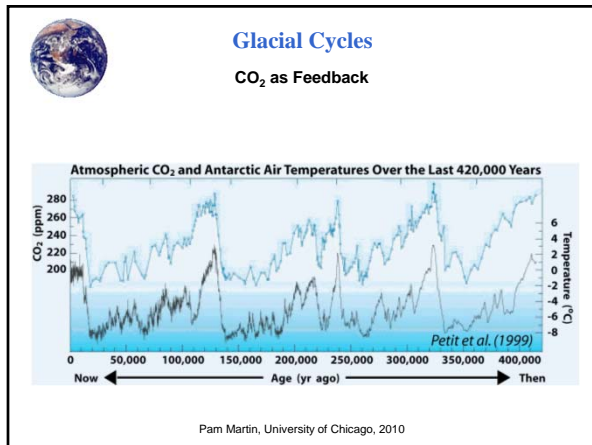
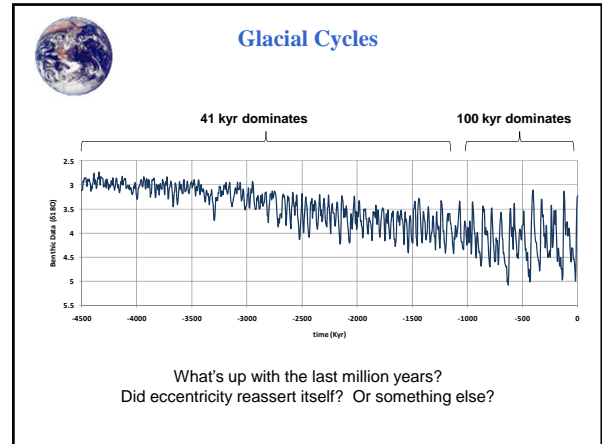
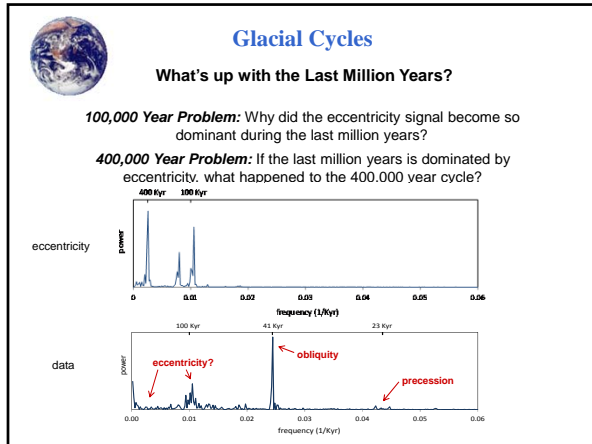
This hypothesis is widely accepted, but also widely regarded as insufficient to explain the observations.

The additional hypothesis is that there are feedback and triggering mechanisms that amplify the Milankovitch cycles. What these mechanisms are and how they work are not fully understood.









**Glacial Cycles**

**Hogg's Model**

Hogg's model shows how the carbon cycle can act as a feedback amplifying and modifying the insolation forcing, but the forcing is somewhat artificial, and the triggering mechanism is difficult to justify.

*What if the 100,000 year glacial cycle is not driven by eccentricity, but is a natural oscillation of the Earth's climate?*

Saltzman and Maasch suggested just such a model.

**Glacial Cycles**

**Salzman-Maasch Model**

Milankovitch forcing

global ice mass  $\rightarrow \dot{X} = -X - Y - uM(t)$

atmospheric CO<sub>2</sub>  $\rightarrow \dot{Y} = -pZ + rY + sZ^2 - Z^2Y$

deep ocean temperature  $\rightarrow \dot{Z} = -q(X + Z)$

Barry Saltzman and Kirk A. Maasch, "A Low-Order Dynamical Model of Global Climatic Variability Over the Full Pleistocene," *Journal of Geophysical Research* 95 (D2), 1955-1963 (1990)

**Glacial Cycles**

**Salzman-Maasch Model**

unforced

Fig. 7. Unforced (free) solution for (4)–(6) with  $q = 1.2$ ,  $s = 0.8$ , and  $p$  and  $r$  varying linearly between  $0.8 \rightarrow 1.0$  and  $0.7 \rightarrow 0.8$ , respectively. (Top) Model solutions for  $X$  (solid) and  $Y$  (dashed). (Bottom) Model solution for  $Z$ .

**Glacial Cycles**

**Salzman-Maasch Model**

forced

Fig. 8. Forced solution for (4)–(6) with  $q = 1.2$ ,  $s = 0.8$ ,  $u = 0.7$ , and  $p$  and  $r$  varying linearly between  $0.8 \rightarrow 1.0$  and  $0.7 \rightarrow 0.8$ , respectively. (Top) Normalized 65°N July insolation curve ( $M$ ) used as forcing. (Middle) Model solutions for  $X$  (solid) and  $Y$  (dashed). (Bottom) Model solution for  $Z$ .

**Glacial Cycles**

**Salzman-Maasch Model**

The Salzman-Maasch model shows how the carbon cycle and the ocean currents can interact to produce unforced oscillations with periods of about 100,000 years. The same model with slightly different parameters can exhibit stationary behavior. By forcing the model with Milankovitch cycles and by slowly varying the parameters over the last two million years, they can produce a bifurcation from small oscillations tracking the Milankovitch cycles to large oscillations with a dominant 100,000 year period.

*Seems like a nice idea, but it is not widely accepted as the explanation, and it has some problems.*

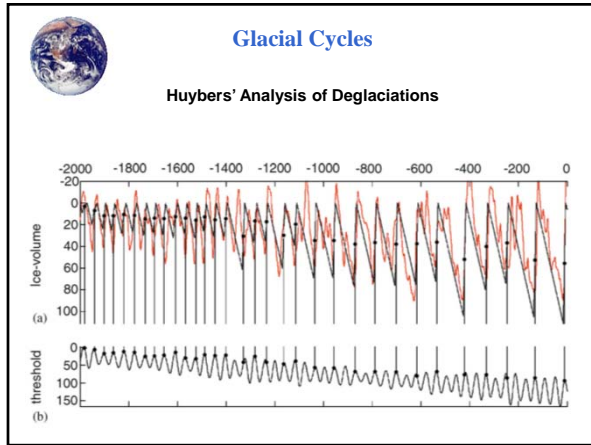
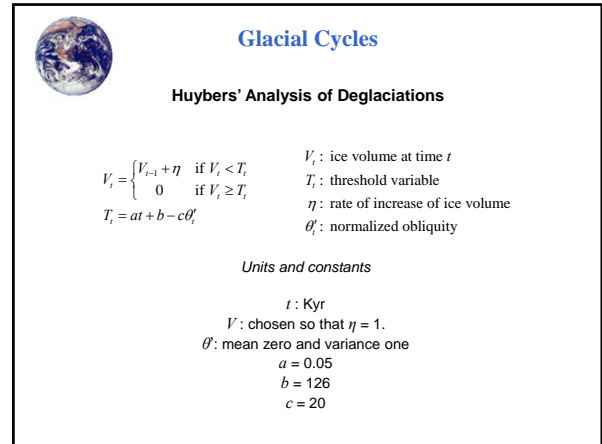
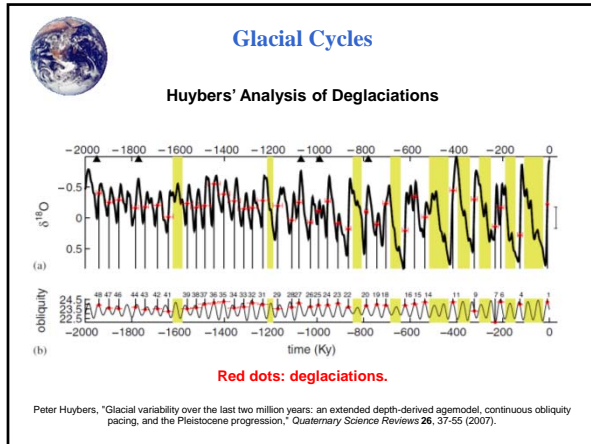
**Glacial Cycles**

**Salzman-Maasch Model**

The Hopf bifurcation explanation seems to have two serious problems ("cosmic coincidences").

1. Why does the intrinsic period of the glacial cycles just happen to have the same period as the eccentricity cycles?
2. Why does the phase of the glacial cycles agree with the phase of the obliquity and eccentricity cycles?

*Ask Samantha.*



**Glacial Cycles**

**Huybers' Analysis of Deglaciations**

Huybers' model produces the decline in temperature and the increase in period and amplitude of the glacial cycles, but it depends heavily on an unspecified decline in the sensitivity of the triggering mechanism over last two million years.

Revised in 2011.

**Glacial Cycles**

**Huybers' 2011 Analysis of Deglaciations**

The deglaciations are triggered by the following forcing function.

$$F_t = \alpha^{1/2} e_t \sin(\rho_t - \phi) + (1 - \alpha)^{1/2} \beta_t$$

where

$e_t$  = eccentricity  
 $\rho_t$  = precession angle  
 $\beta_t$  = obliquity  
 $\alpha$  and  $\phi$  are parameters.

**Glacial Cycles**

**Huybers' 2011 Analysis of Deglaciations**

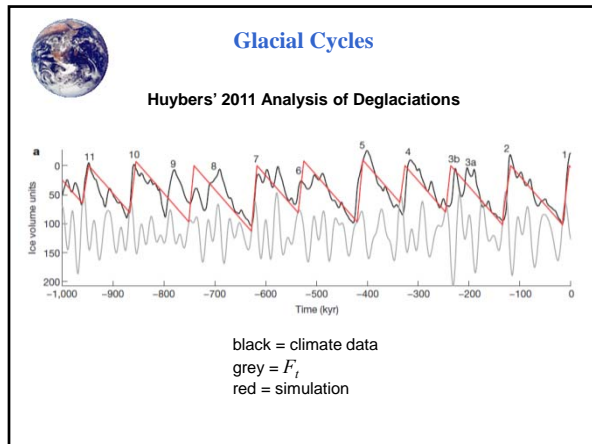
$$V_t = \begin{cases} V_{t-1} + \eta & \text{if } V_t < T_t \\ 0 & \text{if } V_t \geq T_t \end{cases}$$

$T_t = 110 - 25F_t$

$V_t$ : ice volume at time  $t$   
 $T_t$ : threshold variable  
 $\eta$ : rate of increase of ice volume

$$F_t = \alpha^{1/2} e_t \sin(\rho_t - \phi) + (1 - \alpha)^{1/2} \beta_t$$





**Glacial Cycles**

**Abe-Ouchi et al Ice Sheet Model**

The larger the ice sheet, the more unstable it becomes, and the more sensitive it is to insolation. Once it begins to retreat, feedbacks cause a rapid pace.

Ayako Abe-Ouchi, Fuyuki Saito, Kenji Kawamura, Maureen E. Raymo, Jun'ichi Okuno, Kunio Takahashi & Heinz Blatter, "Insolation-driven 100,000-year glacial cycles and hysteresis of ice-sheet volume," *Nature* **500** (2013), 190-193. doi:10.1038/nature12374

Animation available on *Nature* Web site:  
[http://www.nature.com/nature/journal/v500/n7461/fig\\_tab/nature12374\\_SV1.html](http://www.nature.com/nature/journal/v500/n7461/fig_tab/nature12374_SV1.html)

- 
- Glacial Cycles**
- Questions**
1. Did eccentricity play any role during the last million years?  
Is the apparent 100 kyr cycle an artifact (Huybers)?  
Is it an intrinsic cycle in the climate system that coincidentally has a period of 100,000 years (Maasch and Saltzman)?
  2. Is the  $\text{CO}_2$  feedback sufficient to explain the increasing amplitude and period of the glacial cycles during the last million years, *i.e.*, is it the mechanism behind the Huybers model.
  3. Where does the atmospheric  $\text{CO}_2$  go during the glacial maxima?  
The ocean? The land?
  4. What will be the effect of the anthropogenic  $\text{CO}_2$ ?

**Math and Climate Seminar** **IMA**

**MCRN**  
Mathematics and Climate Research Network

**Joint MCRN/IMA Math and Climate Seminar**  
Tuesdays 11:15 – 12:05  
streaming video available at  
[www.ima.umn.edu](http://www.ima.umn.edu)

MCRN [www.mathclimate.org](http://www.mathclimate.org)