Solutions to Homework 5

FM 5021 Mathematical Theory Applied to Finance

7.1 Companies A and B have been offered the following rates per annum on a $20 million 5-year loan:

	Fixed rate	*Floating rate*
Company A:	12.0%	LIBOR + 0.1%
Company B:	13.4%	LIBOR + 0.6%

Company A requires a floating-rate loan; company B requires a fixed-rate loan. Design a swap that will net a bank, acting as intermediary, 0.1% per annum and that will appear equally attractive to both companies.

Company B pays 1.4% per annum more than company A in the fixed-rate markets and only 0.5% per annum more than company A in the floating-rate markets. Therefore, company B has a comparative advantage in floating-rate markets but wants to borrow from the fixed-rate markets. Company A, on the other hand, has a comparative advantage in fixed-rate markets, but wants to borrow from the floating-rate markets. This leads to a swap being negotiated.

The differential between the fixed rates offered to the two companies is 1.4% per annum, and the differential in the floating rates offered to the two companies is 0.5% per annum, so that the total gain to all parties from the swap is $1.4\% - 0.5\% = 0.9\%$ per annum. The bank would get 0.1% per annum, and since the swap should be equally attractive to both companies, each of them should be better of by 0.4% per annum. This means that company A will borrow at $\text{LIBOR} + 0.1\% - 0.4\% = \text{LIBOR} - 0.3\%$ and company B will borrow at $13.4\% - 0.4\% = 13\%$ per annum.

Company A has these three sets of interest rate cash flows:

1. It pays 12% per annum to its outside lenders.
2. It receives 12.3% per annum from the bank.
3. It pays LIBOR to the bank.

Company B has these three sets of interest rate cash flows:

1. It pays LIBOR +0.6% to its outside lenders.
2. It receives LIBOR from the bank.
3. It pays 12.4% to the bank.

The swap agreement is illustrated schematically below.

7.2 Company X wishes to borrow US dollars at a fixed rate of interest. Company Y wishes to borrow Japanese yen at a fixed rate of interest. The

amounts required by the two companies are roughly the same at the current exchange rate. The companies have been quoted the following interest rates, which have been adjusted for the impact of taxes:

	Yen	*Dollars*
Company X:	5.0%	9.6%
Company Y:	6.5%	10.0%

Design a swap that will net a bank, acting as intermediary, 50 basis points per annum. Make the swap equally attractive to the two companies and ensure that all foreign exchange risk is assumed by the bank.

Company Y pays 1.5% more than company X for borrowing yen and only 0.4% per annum more than company A for borrowing dollars. Therefore, company Y has a comparative advantage in the dollar market, but wants to borrow yen, while company X has a comparative advantage in the yen market, but wants to borrow dollars. This leads to a swap being negotiated.

The differential betweeen the yen rates is 1.5% per annum and the differential between the dollar rate is 0.4% per annum, so that the total gain from the swap to all parties is $1.5\% - 0.4\% = 1.1\%$ per annum. The bank gets 0.5% per annum from the total gain, and since the swap should be equally attractive to both companies, each of them should get 0.3% per annum from the total gain. This means that company X will borrow dollars at $9.6\% - 0.3\% = 9.3\%$ per annum and company Y will borrow yen at $6.5\% - 0.3\% = 6.2\%$ per annum. All foreign exchange rate is assumed by the bank.

Company X has these three interest rate cash flows:

1. It pays 5% per annum for yen to its outside lenders.
2. It receives 5% per annum for yen from the bank.
2. It pays 9.3% per annum for dollars to the bank.

Company Y has these three interest rate cash flows:

1. It pays 10% per annum for dollars to its outside lenders.
2. It receives 10% per annum for dollars from the bank.
3. It pays 6.2% per annum for yen to the bank.

The swap is illustrated schematically below.

7.3 A \$100 million interest rate swap has a remaining life of 10 months. Under the terms of the swap, 6-month LIBOR is exchanged for 12% per annum (compounded semiannually). The average of the bid-offer rate being exchanged for 6-month LIBOR in swaps of all maturities is currently 10% per annum with continuous compounding. The 6-month LIBOR rate was 9.6% per annum 2 months ago. What is the current value of the swap to the party paying floating? What is its value to the party paying fixed?

We can value interest rate swaps in terms of bond prices or in terms of FRAs. For this

problem I will use the first method. We first have to value the fixed-rate and the floating-rate bonds underlying the swap. The values of these bonds are the sums of the discounted present values of their cash flows.
The first cash flow received on the fixed-rate bond is $0.5 \times 0.12 \times \$100$ million $= \$6$ million. In 10 months another \$6 million plus the notional principal of \$100 million will be received. Therefore, the value of the fixed-rate bond underlying the swap is

$$(\$6 \text{ million})e^{-0.1\times\frac{4}{12}} + (\$106 \text{ million})e^{-0.1\times\frac{10}{12}} = \$103.328 \text{ million}.$$

The floating payment that will be made in four months is $0.5 \times 0.096 \times \$100$ million $= \$4.8$ million. Therefore, the value of the floating-rate bond underlying the swap is

$$(\$100 \text{ million } + \$4.8 \text{ million })e^{-0.1\times\frac{4}{12}} = \$101.364 \text{ million}.$$

The value of the swap to the party paying the floating rate is \$103.328 million $-$ \$101.364 million $=$ \$1.964 million, and the value of the swap to the party paying the fixed rate is $-$\$1.964 million.

7.4 Explain what a swap rate is. What is the relationship between swap rates and par yields?
The swap rate for a particular maturity is the average of the bid and offer fixed rates that a market maker is prepared to exchange for LIBOR in a standard plain vanilla swap with that maturity. The swap rate for a particular maturity is the LIBOR/swap par yield for the maturity. The swap rate can also be defined as the fixed rate in an interest rate swap that causes the swap to have a value of zero.

7.9 Companies X and Y have been offered the following rates per annum on a \$5 million 10-year investment:

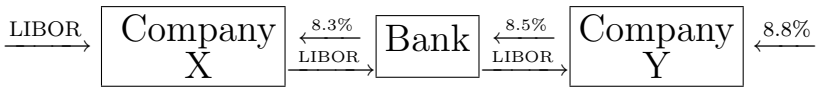
	Fixed rate	*Floating rate*
Company X:	8.0%	LIBOR
Company Y:	8.8%	LIBOR

Company X requires a fixed-rate investment; company Y requires a floating-rate investment. Design a swap that will net a bank, acting as intermediary, 0.2% per annum and will appear equally attractive to X and Y.
The differential between the fixed rates offered to the two companies is 0.8% per annum, and the differential between the floating rates offered is 0.0% per annum, so that the total gain to all parties from the swap is 0.8% per annum. The bank will get 0.2% per annum, and since the swap should be equally attractive to both companies, each of them should get 0.3% per annum from the total gain. Therefore, company A should be able to get a fixed-rate return of 8.3% per annum, while company B should be able to get a floating-rate return of LIBOR +0.3% per annum.
Company X has the following interest rate cash flows:

1. It receives LIBOR for its investment.
2. It pays LIBOR to the bank.
3. It receives 8.3% per annum from the bank.

Company Y has the following interest rate cash flows:
1. It receves 8.8% per annum for its investment.
2. It pays 8.5% per annum to the bank.
3. It receives LIBOR from the bank.

The swap is illustrated schematically below.



7.15 Why is the expected loss from a default on a swap less than the expected loss from the default on a loan with the same principal?

The expected loss from a default on a swap is less than the expected loss from the default on a loan with the same principal, because a financial institution has no exposure to the notional principal in an interest rate swap. In an interest rate swap, the financial institution's exposure depends only on the difference between a fixed rate of interest and a floating rate of interest. In a loan the whole principal can be lost.