

**Math 2263.002**  
Summer 2009  
Exam 1

Name:

There are six numbered problems, each worth 25 points, on three sheets of paper.

1. Write parametric equations for the line of intersection of the planes

$$x - y + 2z = 3$$

and

$$z - x + 2 = 0.$$

2. Find the limit, if it exists, or show that the limit does not exist:

$$\lim_{(x,y) \rightarrow (0,0)} \frac{5x^2y}{x^3 + y^3}.$$

3. Let  $f(x, y) = 3x^2 - (xy)^2 + 2x$ .

(a) (15) Find an equation of the tangent plane to the graph of  $f$  when  $(x, y) = (1, -2)$ .

(b) (10) Find an equation of the plane passing through the origin that is parallel to the plane in (a).

4. Let  $f(x, y) = x^2 \sin y + ye^{xy}$  and

$$x(s, t) = s + 2t,$$

$$y(s, t) = st.$$

(a) (15) Use the chain rule to write  $\frac{\partial f}{\partial t}$ . You can write your answer in terms of  $x$ ,  $y$ ,  $s$  and  $t$ .

(b) (10) Evaluate  $\frac{\partial f}{\partial t}$  at  $(s, t) = (2, 1)$ . Write an *exact* number, not a decimal approximation.

5. Let

$$f(x, y, z) = \frac{x - y}{z^2}.$$

(a) (6) Find the gradient of  $f$ .

(b) (10) Find the directional derivative of  $f$  at the point  $(-2, 3, -1)$  in the direction of the vector  $\langle 2, 1, 3 \rangle$ .

(c) (9) Find the maximum rate of change of  $f$  at the point  $(-2, 3, -1)$ , and write a *unit* vector indicating the direction in which that maximum rate of change occurs.

6. Let

$$f(x, y) = 3xy - x^2y - xy^2.$$

- (a) (15) Find the local maximum and minimum values and saddle points of  $f$ .

- (b) (10) Find the absolute maximum and minimum values of  $f$  on the triangular region defined by the inequalities  $x \geq 0$ ,  $y \geq 0$ , and  $x + y \leq 3$ .