Math 2263.002
Summer 2009
Exam 1 solutions

The exam is worth 150 points. Thirty-six students took the exam. The mean was 107.4 (71.6%), and the median was 114.5 (76.3%).

1. $\mathbf{n_1} = \langle 1, -1, 2 \rangle$ **(5 points)**. $\mathbf{n_2} = \langle -1, 0, 1 \rangle$ **(5)**. $\mathbf{n_2} \times \mathbf{n_1} = \langle 1, 3, 1 \rangle$ **(5)**. The point $(2, -1, 0)$ is on the line of intersection **(5)**. $\mathbf{r}(t) = \langle 2, -1, 0 \rangle + t \langle 1, 3, 1 \rangle$. $x(t) = 2 + t$, $y(t) = -1 + 3t$, and $z(t) = t$ **(5)**.

2. Along $y = 0$, $\frac{5x^2y}{x^3+y^3} = \frac{0}{x^3} = 0$, when $x \neq 0$ **(5)**. Along $y = x$, $\frac{5x^2y}{x^3+y^3} = \frac{5x^3}{2x^3} = \frac{5}{2}$, when $(x, y) \neq (0, 0)$ **(5)**. Thus, the limit does not exist. **(5)**.

3. (a) $f(1, -2) = 1$. $f_x = 6x - 2xy^2 + 2$ **(3)**. $f_x(1, -2) = 0$ **(3)**. $f_y = -2x^2y$ **(3)**. $f_y(1, -2) = 4$ **(3)**. An equation for the tangent plane is $z - 1 = 4(y + 2)$ **(3)**.

   (b) $\mathbf{n} = \langle 0, 4, -1 \rangle$ **(5)**. An equation is $z = 4y$ **(5)**.

4. (a) $\frac{\partial f}{\partial t} = \frac{\partial f}{\partial x}\frac{\partial x}{\partial t} + \frac{\partial f}{\partial y}\frac{\partial y}{\partial t}$ **(3)** $= (2x \sin y + y^2 e^{xy})(2) + (x^2 \cos y + e^{xy} + xye^{xy})(s)$ **(3 for each of 4 factors)**.

   (b) $(x, y) = (4, 2)$ **(2)**. $\frac{\partial f}{\partial t}(2, 1) = (2)(8 \sin 2 + 4e^8) + (2)(16 \cos 2 + e^8 + 8e^8)$ **(4 for each of 2 summands)** $= 16 \sin 2 + 26e^8 + 32 \cos 2$.

5. (a) $\nabla f = \left\langle z^{-2}, -z^{-2}, -2(x - y)z^{-3} \right\rangle$ **(2 for each entry)**.

   (b) $\nabla f(-2, 3, -1) = \langle 1, -1, -10 \rangle$ **(2)**. $\mathbf{u} = \frac{1}{\sqrt{14}} \langle 2, 1, 3 \rangle$ **(2)**. $D_{\mathbf{u}} f(-2, 3, -1) = \nabla f(-2, 3, -1) \cdot \mathbf{u}$ **(2)** $= \langle 1, -1, -10 \rangle \cdot \frac{1}{\sqrt{14}} \langle 2, 1, 3 \rangle = \frac{-29}{\sqrt{14}}$ **(4)**.

   (c) The maximum rate of change is $|\nabla f(-2, 3, -1)| = \sqrt{102}$ **(6)**, and it occurs in the direction of the unit vector $\frac{1}{\sqrt{102}} \langle 1, -1, -10 \rangle$ **(3)**.

6. (a) $f_x = 3y - 2xy - y^2$ and $f_y = 3x - x^2 - 2xy$ **(2 each)**. There are four critical points: $(0, 0)$, $(3, 0)$, and $(0, 3)$ are all saddle points, and there is a local max at $(1, 1)$. **(2 for each CP and 2 for each classification)**.

   (b) $f = 0$ along the entire boundary, including the hypotenuse **(5)**. That is the minimum value **(5)**, and the maximum is $f(1, 1) = 1$ **(5)**.