**Math 2263.002** Name:
Summer 2009
Exam 2

There are six numbered problems on three sheets of paper. Point values are in parentheses.

1. (26) Let $E$ be the solid bounded by the four planes $2x + 3y + 6z = 12$, $x = 3$, $y = 2$, and $z = 1$.

   (a) (13) Write a triple integral, with integration in the order $dz\,dx\,dy$, that represents the volume of the solid $E$. Include correct integrand and limits of integration. Do not evaluate the integral.

   (b) (13) Write a triple integral, with integration in the order $dx\,dy\,dz$, that represents the volume of the solid $E$. Include correct integrand and limits of integration. Do not evaluate the integral.

2. (26) Let $f(x, y) = xy^2$ and let $R = [2, 10] \times [1, 5]$.

   (a) (14) Estimate the value of the integral

   $$\iint_R f(x, y)\, dA$$

   by subdividing the $x$-interval $[2, 10]$ into 2 subintervals and subdividing the $y$-interval $[1, 5]$ into 2 subintervals and evaluating $f(x, y)$ at the midpoint of each subrectangle.

   (b) (12) Find the exact value of

   $$\iint_R f(x, y)\, dA.$$

3. (25) Find the volume of the solid bounded by the hyperboloid $z^2 - 1 = x^2 + y^2$ and the plane $z = 3$.

4. (25) Applications.

   (a) Probability. Suppose that the joint density function for a pair of random variables $X$ and $Y$ is

$$f(x,y) = \begin{cases} C, & \text{if } 0 \le x \le 10 \text{ and } 0 \le y \le 20 \\ 0, & \text{otherwise,} \end{cases}$$

   where $C$ is a constant.

      i. (5) Find the value of $C$.

      ii. (5) Find $P(X + Y \ge 20)$.

   (b) (15) Center of mass. Suppose that a flat plate is described as the region bounded between the curves $y = \frac{1}{2}x^2$ and $y = 3x - 4$. Suppose also that the density of the plate is constant, $\delta(x, y) = 6$ kg/m$^2$, and as a result the mass of the plate is 4 kg.

   Write an expression for $\bar{y}$, the $y$-coordinate of the center of mass, that involves an integral with correct integrand and limits of integration. Do not evaluate the integral.

5. (24) Write a double integral using polar coordinates that represents the area of the region in the first quadrant that lies outside the polar curve $r = 1 + \cos\theta$ and inside the polar curve $r = 1 + \sin\theta$. Include correct integrand and limits of integration. Do not evaluate the integral.

6. (24) Let $f(x, y, z) = x^2 y$ and let $g(x, y, z) = 32x^2 + 2y^2 + 8z^2$.

   (a) (12) Find $\nabla f$ and $\nabla g$.

   (b) (12) Find the maximum and minimum values of the function

   $$f(x, y, z) = x^2 y$$

   subject to the constraint that

   $$g(x, y, z) = 32x^2 + 2y^2 + 8z^2 = 32,$$

   and write the point(s) at which those minimum and maximum values occur.