Math 2263.002
Summer 2009
Exam 2 solutions

The exam is worth 150 points. Thirty-seven students took the exam. The mean was 109.2 (72.8%), and the median was 107 (71.3%). Point values in brackets.

1. (a)
$$\int_0^2 \int_{\frac{6-3y}{2}}^3 \int_{\frac{12-2x-3y}{6}}^1 1\, dz\, dx\, dy.$$
(b)
$$\int_0^1 \int_{\frac{6-6z}{3}}^2 \int_{\frac{12-3y-6z}{2}}^3 1\, dx\, dy\, dz.$$
[For each part, 2 pts. per limit and 1 pt. for integrand.]

2. (a)
$$\iint_R f\, dA \approx (f(4,2) + f(8,2) + f(4,4) + f(8,4))\Delta x \Delta y$$
$$= (16 + 32 + 64 + 128) \cdot 4 \cdot 2 = 1920.$$
[2 pts. each for: each function value, Δx, Δy, and answer.]
(b)
$$\iint_R f\, dA = \int_1^5 \int_2^{10} xy^2\, dx\, dy \qquad [2]$$
$$= \left[\frac{x^2}{2}\right]_2^{10} \cdot \left[\frac{y^3}{3}\right]_1^5 \qquad [2 \text{ each}]$$
$$= \frac{1}{6}(100 - 4) \cdot (125 - 1) \qquad [2 \text{ each eval.}]$$
$$= 1984. \qquad [2]$$

3. The surfaces intersect when $x^2 + y^2 = 8$. [5]
$$V = \iint_R (3 - \sqrt{1 + x^2 + y^2})\, dA \qquad [5]$$
$$= \int_0^{2\pi} \int_0^{\sqrt{8}} (3 - \sqrt{1 + r^2}) r\, dr\, d\theta \qquad [5]$$
$$= \frac{20\pi}{3}.$$
[5 each for evaluation and answer]

4. (a) i.

$$1 = \iint_R f\, dA = 200C \implies C = \frac{1}{200}. \qquad [5]$$

ii. Since the density function is constant, and the area of the resulting triangle is one-fourth that of the rectangle, we have $P = 1/4$. [5]

(b)

$$\bar{y} = \frac{1}{4} \int_2^4 \int_{\frac{1}{2}x^2}^{3x-4} 6y\, dy\, dx.$$

[3 each for pairs of limits and the numbers $\frac{1}{4}$, 6, and y.]

5.

$$A = \int_{\frac{\pi}{4}}^{\frac{\pi}{2}} \int_{1+\cos\theta}^{1+\sin\theta} 1 \cdot r\, dr\, d\theta.$$

[4 each for: $\frac{\pi}{2}$, $\frac{\pi}{4}$, $1+\sin\theta$, $1+\cos\theta$, 1, $r\, dr\, d\theta$.]

6. (a) $\nabla f = \langle 2xy, x^2, 0 \rangle$ and $\nabla g = \langle 64x, 4y, 16z \rangle$. [2 per entry]

(b) $\nabla f = \lambda \nabla g$ gives $2xy = \lambda \cdot 64x$, $x^2 = \lambda \cdot 4y$ and $0 = \lambda \cdot 16z$. [1 each] If $\lambda = 0$, then $x = 0$, where $f = 0$. Otherwise, $\lambda \neq 0$ and $z = 0$, which implies $32x^2 + 2y^2 = 32$. If one of x and y is 0, then so is the other, a contradiction. So both are nonzero, and by eliminating λ, we get $y = \pm\frac{4}{\sqrt{3}}$ and $x = \pm\sqrt{\frac{2}{3}}$. $f(\pm\sqrt{\frac{2}{3}}, \frac{4}{\sqrt{3}}, 0) = \frac{8}{3\sqrt{3}}$, the maximum, and $f(\pm\sqrt{\frac{2}{3}}, -\frac{4}{\sqrt{3}}, 0) = \frac{-8}{3\sqrt{3}}$, the minimum. [3 each for the max, the min, $\pm\sqrt{\frac{2}{3}}$, $\frac{4}{\sqrt{3}}$ and $-\frac{4}{\sqrt{3}}$.]