**Math 2263.002** Name:
Summer 2009
Exam 3

In all cases where numerical answers are required, give exact answers, not decimal approximations.

1. (24 points; 3 each)

   (a) Answer each question with a short phrase.

      i. What does it mean for a curve $C$ to be *closed*?

      ii. What does it mean for a closed curve $C$ to be *simple*?

      iii. What does it mean for a simple, closed curve $C$ to be *positively-oriented*?

      iv. What does it mean for a vector field $\mathbf{F}$ to be *conservative*?

   (b) Let $\mathbf{F} = \langle y + 2xz, x + 3y^2z, x^2 + y^3 \rangle$.

      i. Find $\operatorname{curl} \mathbf{F}$.

      ii. Find $\operatorname{div} \mathbf{F}$.

      iii. Is the vector field $\mathbf{F}$ conservative? Give a short justification.

      iv. Is there a vector field $\mathbf{G}$ such that $\mathbf{F} = \operatorname{curl} \mathbf{G}$? Give a short justification.

2. (26) Let $\mathbf{F} = \left\langle y + e^{x^2}, x^2 + e^{y^2} \right\rangle$, and let $C$ be the positively-oriented boundary of the triangle that has vertices $(1, 1)$, $(3, 1)$, and $(1, 3)$. Find $\int_C \mathbf{F} \cdot d\mathbf{r}$.

3. (25) Find the mass of the solid that lies in the first octant, is trapped between the spheres

$$x^2 + y^2 + z^2 = 16$$

and

$$x^2 + y^2 + z^2 = 25,$$

and lies above the top half of the cone $3z^2 = x^2 + y^2$ if the density of the solid, in grams per cubic centimeter, is given at each point by the distance between the point and the origin.

4. (25) Let $\mathbf{F} = \langle -z \sin x \sin y, z \cos x \cos y, \cos x \sin y \rangle$, and let $C$ be the curve from $(0, 3, 5)$ to $(0, -3, 5)$ that traces a semi-circle in the $yz$-plane given by $y^2 + (z-5)^2 = 9$, $z \geq 5$. Find

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

5. (25) Let

$$\mathbf{F} = \left\langle z, e^{\cos(xz)^2}, xy \right\rangle,$$

and let $C$ be the directed line segment from $(2, 1, -3)$ to $(-1, 1, 4)$. Find

$$\int_C \mathbf{F} \cdot d\mathbf{r}.$$

6. (25) Find the mass of a wire represented by the graph of the function $y = x^3$ between the points $(1, 1)$ and $(3, 27)$, if the density of the wire, in grams per centimeter, at each point is given by the cube of the distance from the point to the $y$-axis.