

**Math 2263.002**

Summer 2009

Exam 3 solutions

1. (24 points; 3 each)
  - (a)
    - i. The endpoints are the same.
    - ii. The curve intersects itself only at the endpoints.
    - iii. It travels counter-clockwise around the region it bounds; or, the region is on the left as it travels.
    - iv. There is a function  $f$  such that  $\nabla f = \mathbf{F}$ .
  - (b)
    - i.  $\langle 3y^2 - 3y^2, 2x - 2x, 1 - 1 \rangle = \vec{0}$ .
    - ii.  $2z + 6yz$
    - iii. Yes, since  $\text{curl } \mathbf{F} = \vec{0}$ .
    - iv. No, since  $\text{div } \mathbf{F} \neq 0$ .
2. (26) By Green's theorem,  $\int_C \mathbf{F} \cdot d\mathbf{r} = \iint_R (Q_x - P_y) dA$

$$= \int_1^3 \int_1^{4-x} (2x - 1) dy dx = \frac{14}{3}.$$

3. (25) Between the spheres, we have  $4 \leq \rho \leq 5$ . In the first octant, we have  $0 \leq \theta \leq \frac{\pi}{2}$ . Above the top half of the cone, we have  $0 \leq \phi \leq \frac{\pi}{3}$ .

$$\begin{aligned} M &= \int_0^{\frac{\pi}{3}} \int_0^{\frac{\pi}{2}} \int_4^5 \rho(\rho^2 \sin \phi) d\rho d\theta d\phi \\ &= \frac{5^4 - 4^4}{4} \cdot \frac{\pi}{2} \cdot \left(-\frac{1}{2} + 1\right) = \frac{\pi}{16}(5^4 - 4^4). \end{aligned}$$

4. (25)  $\nabla f = \mathbf{F}$ , where  $f = z \cos x \sin y$ . By the fundamental theorem for line integrals,  $\int_C \mathbf{F} \cdot d\mathbf{r} = f(0, -3, 5) - f(0, 3, 5) = 5 \sin(-3) - 5 \sin 3 = -10 \sin 3$ .
5. (25) Parametrize  $C$ :  $x(t) = 2 - 3t$ ,  $y(t) = 1$ , and  $z(t) = -3 + 7t$ , for  $0 \leq t \leq 1$ . Then

$$\int_C \mathbf{F} \cdot d\mathbf{r} = \int_0^1 \langle -3 + 7t, \dots, 2 - 3t \rangle \cdot \langle -3, 0, 7 \rangle dt = 2.$$

6. (25) Parametrize  $C$  by  $x(t) = t$ ,  $y(t) = t^3$ , for  $1 \leq t \leq 3$ . Then

$$M = \int_1^3 t^3 \sqrt{1 + 9t^4} dt = \frac{1}{54}(730^{3/2} - 10^{3/2}).$$