Clusters-Jan2011

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```
attach /home/faculty/musiker/SAGE/clusters-lacim-queue/cluster-nb.sage
```

```
S = ClusterSeed(['R2',[1,1],2]); S
```

A seed for a cluster algebra of rank 2 of type ['A', 2]

```
S.cluster();
```

$[x_0, x_1]$

```
S.b_matrix()
```

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

```
S.mutate(0); S.cluster()
```

$\left[\frac{x_1+1}{x_0}, x_1\right]$

```
S.b_matrix()
```

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

```
S.mutate(1); S.cluster()
```

$\left[\frac{x_1+1}{x_0}, \frac{x_0+x_1+1}{x_0x_1}\right]$

```
S.b_matrix()
```

$$\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}$$

```
S.mutate(0); S.cluster()
```

$\left[\frac{x_0+1}{x_1}, \frac{x_0+x_1+1}{x_0x_1}\right]$

```
S.b_matrix()
```

$$\begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}$$

```
S.mutate(1); S.cluster()
```

$\left[\frac{x_0+1}{x_1}, x_0\right]$

```
S.mutate(0); S.cluster()
```

$[x_1, x_0]$

```
S.variable_class()
```

$\left[\frac{x_1+1}{x_0}, x_1, \frac{x_0+1}{x_1}, \frac{x_0+x_1+1}{x_0x_1}, x_0\right]$

```
S2 = ClusterSeed(['R2',[1,2],2]); S2
```

A seed for a cluster algebra of rank 2 of type ['B', 2]

```
S2.cluster()
```

$$[x_0, x_1]$$

```
S2.b_matrix()
```

$$\begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}$$

```
S2.mutate(0); S2.cluster()
```

$$\left[\frac{x_1^2+1}{x_0}, x_1\right]$$

```
S2.b_matrix()
```

$$\begin{pmatrix} 0 & 1 \\ -2 & 0 \end{pmatrix}$$

```
S2.mutate(1); S2.cluster()
```

$$\left[\frac{x_1^2+1}{x_0}, \frac{x_1^2+x_0+1}{x_0x_1}\right]$$

```
S2.mutate([0,1]); S2.cluster()
```

$$\left[\frac{x_0^2+x_1^2+2x_0+1}{x_0x_1^2}, \frac{x_0+1}{x_1}\right]$$

```
S2.mutate([0,1]); S2.cluster()
```

$$[x_0, x_1]$$

```
S2.variable_class()
```

$$\left[\frac{x_1^2+1}{x_0}, \frac{x_0+1}{x_1}, \frac{x_0^2+x_1^2+2x_0+1}{x_0x_1^2}, x_1, x_0, \frac{x_1^2+x_0+1}{x_0x_1}\right]$$

```
S3 = ClusterSeed(['R2',[2,2],2]); S3
```

A seed for a cluster algebra of rank 2 of type ['A', [1, 1], 1]

```
S3.mutate([0,1]); S3.cluster()
```

$$\left[\frac{x_1^2+1}{x_0}, \frac{x_1^4+x_0^2+2x_1^2+1}{x_0^2x_1}\right]$$

```
S3.mutate([0,1]); S3.cluster()
```

$$\left[\frac{x_1^6+x_0^4+2x_0^2x_1^2+3x_1^4+2x_0^2+3x_1^2+1}{x_0^3x_1^2}, \frac{x_1^8+x_0^6+2x_0^4x_1^2+3x_0^2x_1^4+4x_1^6+3x_0^4+6x_0^2x_1^2+6x_1^4+3x_0^2+4x_1^2+1}{x_0^4x_1^3}\right]$$

```
S3.mutate([0,1]); S3.cluster()
```

$$\left[\frac{x_1^{10}+x_0^8+2x_0^6x_1^2+3x_0^4x_1^4+4x_0^2x_1^6+5x_1^8+4x_0^6+9x_0^4x_1^2+12x_0^2x_1^4+10x_1^6+6x_0^4+12x_0^2x_1^2+10x_1^4+4x_0^2+5x_1^2+1}{x_0^5x_1^4}, \frac{x_1^{12}+x_0^{10}+2x_0^8x_1^2+3x_0^6x_1^4+4x_0^4x_1^6+5x_0^2x_1^8+6x_1^{10}+5x_0^8+12x_0^6x_1^2+18x_0^4x_1^4+20x}{}\right]$$

```
S3.variable_class()
```

```
Traceback (click to the left of this block for traceback)
...
AssertionError: The variable class can - for infinite types - only be
computed up to a given depth
```

```
S3.set_cluster([1,1])
```

```
S3.mutate([0,1]); S3.cluster()
```

$$[2, 5]$$

```
S3.mutate([0,1]); S3.cluster()
```

$[13, 34]$

```
S3.mutate([0,1]); S3.cluster()
```

$[89, 233]$

```
Mat4 = matrix([[0,-1,2,-1],[1,0,-3,2],[-2,3,0,-1],[1,-2,1,0]]); Mat4
```

$$\begin{bmatrix} 0 & -1 & 2 & -1 \\ 1 & 0 & -3 & 2 \\ -2 & 3 & 0 & -1 \\ 1 & -2 & 1 & 0 \end{bmatrix}$$

```
Somos4 = ClusterSeed(Mat4); Somos4
```

A seed for a cluster algebra of rank 4

```
Somos4.b_matrix()
```

$$\begin{bmatrix} 0 & -1 & 2 & -1 \\ 1 & 0 & -3 & 2 \\ -2 & 3 & 0 & -1 \\ 1 & -2 & 1 & 0 \end{bmatrix}$$

```
Somos4.cluster()
```

$[x_0, x_1, x_2, x_3]$

```
Somos4.mutate([0,1,2,3]); Somos4.cluster()
```

$$\left[\frac{x_2^2+x_1x_3}{x_0}, \frac{x_2^3+x_1x_2x_3+x_0x_3^2}{x_0x_1}, \frac{x_1x_2^4+2x_1^2x_2^2x_3+x_0x_2^3x_3+x_1^3x_3^2+x_0x_1x_2x_3^2+x_0^2x_3^3}{x_0^2x_1x_2}, \frac{x_1^2x_2^6+x_0x_2^7+3x_1^3x_2^4x_3+3x_0x_1x_2^5x_3+3x_1^4x_2^2x_3^2+3x_0x_1^2x_2^3x_3^2+2x_0^2x_2^4x_3^2+x_1^5x_3^3+x_0x_1^3x_2x_3^3+3x_0^2x_1x_2^2x_3^3}{x_0^3x_1^2x_2x_3}\right.$$

```
Somos4.b_matrix()
```

$$\begin{bmatrix} 0 & -1 & 2 & -1 \\ 1 & 0 & -3 & 2 \\ -2 & 3 & 0 & -1 \\ 1 & -2 & 1 & 0 \end{bmatrix}$$

```
Somos4.set_cluster([1,1,1,1]); Somos4.mutate([0,1,2,3]); Somos4.cluster()
```

$[2, 3, 7, 23]$

```
Somos4.mutate([0,1,2,3]); Somos4.cluster()
```

$[59, 314, 1529, 8209]$

```
Somos4.mutate([0,1,2,3]); Somos4.cluster()
```

$[83313, 620297, 7869898, 126742987]$

```
SeedA3 = ClusterSeed(['A',3]); SeedA3
```

A seed for a cluster algebra of rank 3 of type ['A', 3]

```
SeedA3.b_matrix()
```

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

```
SeedA3.cluster()
```

$$[x_0, x_1, x_2]$$

```
SeedA3.mutate(0); SeedA3.cluster()
```

$$\left[\frac{x_1+1}{x_0}, x_1, x_2\right]$$

```
SeedA3.b_matrix()
```

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & -1 \\ 0 & 1 & 0 \end{bmatrix}$$

```
SeedA3.mutate([0,1]); SeedA3.cluster()
```

$$\left[x_0, \frac{x_0x_2+1}{x_1}, x_2\right]$$

```
SeedA3.b_matrix()
```

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

```
SeedA3.mutate([1,2]); SeedA3.cluster()
```

$$\left[x_0, x_1, \frac{x_1+1}{x_2}\right]$$

```
SeedA3.b_matrix()
```

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

```
SeedA3.mutate(0); SeedA3.cluster()
```

$$\left[\frac{x_1+1}{x_0}, x_1, \frac{x_1+1}{x_2}\right]$$

```
SeedA3.b_matrix()
```

$$\begin{bmatrix} 0 & -1 & 0 \\ 1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

```
SeedA3.mutate(1); SeedA3.cluster()
```

$$\left[\frac{x_1+1}{x_0}, \frac{x_1^2+x_0x_2+2x_1+1}{x_0x_1x_2}, \frac{x_1+1}{x_2}\right]$$

```
SeedA3.mutate([1,0]); SeedA3.cluster()
```

$$\left[x_0, x_1, \frac{x_1+1}{x_2}\right]$$

```
SeedA3.b_matrix()
```

$$\begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 1 \\ 0 & -1 & 0 \end{bmatrix}$$

```
SeedA3.mutate(1); SeedA3.cluster()
```

$$\left[x_0, \frac{x_0x_2+x_1+1}{x_1x_2}, \frac{x_1+1}{x_2}\right]$$

```
SeedA3.variable_class()
```

$$\left[x_2, \frac{x_1+1}{x_0}, \frac{x_1^2+x_0x_2+2x_1+1}{x_0x_1x_2}, \frac{x_0x_2+x_1+1}{x_1x_2}, x_1, x_0, \frac{x_0x_2+x_1+1}{x_0x_1}, \frac{x_0x_2+1}{x_1}, \frac{x_1+1}{x_2}\right]$$

```
len(_)
```

9

```
SeedA4 = ClusterSeed(['A',4]); SeedA4
```

A seed for a cluster algebra of rank 4 of type ['A', 4]

```
SeedA4.b_matrix()
```

$$\begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & -1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

```
SeedA4.variable_class()
```

$$\left[\frac{x_1+1}{x_0}, \frac{x_1x_3+x_2+1}{x_2x_3}, x_1, x_0, \frac{x_1^2x_3+x_0x_2+x_1x_3+x_1+1}{x_0x_1x_2}, x_2, \frac{x_0x_2+x_1x_3+1}{x_1x_2}, \frac{x_0x_2^2+x_1^2x_3+x_0x_2+x_1x_2+x_1x_3+x_1+x_2+1}{x_0x_1x_2x_3}, \frac{x_1x_3+1}{x_2}, \frac{x_0x_2^2+x_0x_2+x_1x_3+x_2+1}{x_1x_2x_3}, x_3, \frac{x_0x_2+x_1+}{x_0x_1}\right.$$

```
len(_)
```

14

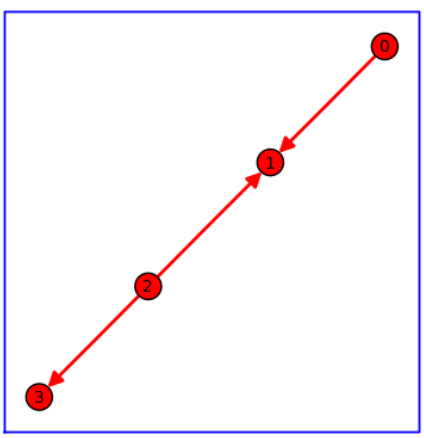
```
LL = map(denominator,SeedA4.variable_class()); LL.sort(); LL
```

$$[1, 1, 1, 1, x_3, x_2, x_1, x_0, x_2x_3, x_1x_2, x_0x_1, x_1x_2x_3, x_0x_1x_2, x_0x_1x_2x_3]$$

```
[monom.degrees() for monom in LL]
```

$$[(0,0,0,0), (0,0,0,0), (0,0,0,0), (0,0,0,0), (0,0,0,1), (0,0,1,0), (0,1,0,0), (1,0,0,0), (0,0,1,1), (0,1,1,0), (1$$

```
SeedA4.show()
```




```
SeedA4.interact()
```

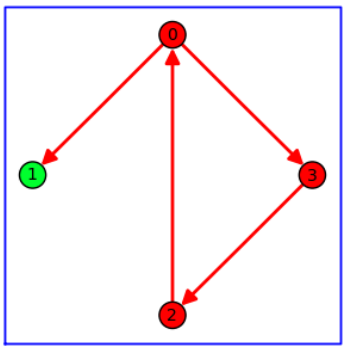
Mutate at: 0 1 2 3

Mutation sequence: ☑

Cluster variables: ☑

B-Matrix: ☑

Show last mutation: ☑



Mutation sequence: 0, 1, 0, 2, 1

Cluster variables:

$$v_0 = \frac{x_0x_2 + 1}{x_1}$$

$$v_1 = x_0$$

$$v_2 = \frac{x_0x_2 + x_1x_3 + 1}{x_1x_2}$$

$$v_3 = x_3$$

B-Matrix:

$$\begin{matrix} 0 & 1 & -1 & 1 \\ -1 & 0 & 0 & 0 \\ 1 & 0 & 0 & -1 \\ -1 & 0 & 1 & 0 \end{matrix}$$

```
SeedD5 = ClusterSeed(['D',5]); SeedD5
```

A seed for a cluster algebra of rank 5 of type ['D', 5]

```
SeedD5.interact()
```

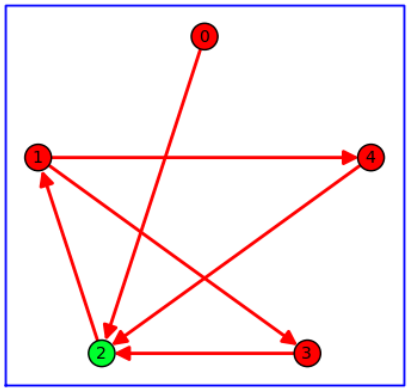
Mutate at: 0 1 2 3 4

Mutation sequence: ☑

Cluster variables: ☑

B-Matrix: ☑

Show last mutation: ☑



Mutation sequence: 1,1,2,2,0,1,2
Cluster variables:

$$v_0 = \frac{x_1 + 1}{x_0}$$
$$v_1 = \frac{x_0x_2 + x_1 + 1}{x_0x_1}$$
$$v_2 = \frac{x_1^2x_3x_4 + x_1x_3x_4 + x_0x_2 + x_1 + 1}{x_0x_1x_2}$$
$$v_3 = x_3$$
$$v_4 = x_4$$

B-Matrix:

$$\begin{matrix} 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & -1 & 1 & 1 \\ -1 & 1 & 0 & -1 & -1 \\ 0 & -1 & 1 & 0 & 0 \\ 0 & -1 & 1 & 0 & 0 \end{matrix}$$

```
SeedD5.variable_class()
```

$$\left[\frac{x_1+1}{x_0}, \frac{x_0x_2^2+x_1x_3x_4+x_0x_2+x_2+1}{x_1x_2x_3}, \frac{x_2+1}{x_4}, x_1, x_0, \frac{x_1x_3x_4+x_2+1}{x_2x_3}, x_3, x_2, \frac{x_0x_2+x_1+1}{x_0x_1}, \frac{x_0x_2^2+x_1^2x_3x_4+2x_0x_2^2+x_1x_2^2+x_1x_3x_4+x_0x_2+2x_1x_2+x_2^2+x_1+2x_2+1}{x_0x_1x_2x_3x_4}, \frac{x_1^2x_3x_4+x_0}{}\right.$$

```
LL2 = map(denominator,_); LL2.sort(); LL2
```

$$[1, 1, 1, 1, 1, x_4, x_3, x_2, x_1, x_0, x_2x_4, x_2x_3, x_1x_2, x_0x_1, x_2x_3x_4, x_1x_2x_4, x_1x_2x_3, x_0x_1x_2, x_1x_2x_3x_4, x_0x_1x_2x_4, x_0x_1x_2x_3, x_1x_2^2x_3x_4,$$

```
[monom.degrees() for monom in LL2]
```

$$[(0,0,0,0,0), (0,0,0,0,0), (0,0,0,0,0), (0,0,0,0,0), (0,0,0,0,0), (0,0,0,0,1), (0,0,0,1,0), (0,0,1,0,0), (0,1,0$$