Nonradial Solutions for a Conformally Invariant Fourth Order Equation in $\mathbb{R}^4$

Ten years ago, C.S. Lin considered the following fourth order elliptic PDE:

$$\Delta^2 u = 6e^{4u} \text{ in } \mathbb{R}^4, \quad \int_{\mathbb{R}^4} e^{4u} dx < \infty. \tag{1}$$

He obtained all possible behavior for solutions of (1) at infinity and classified the solutions if $u(x) = o(|x|^2)$ when $|x| \to \infty$. With J.C. Wei, we prove the complete converse of Lin's result, which shows that there are many nonradial solutions of (1). Geometrically, it means that there exists a very rich family of conformal transformations with constant $Q-$curvature on $\mathbb{R}^4$. This is a striking difference with the two dimensional analogue for the Gauss curvature.