

Solutions for Worksheet 7923 Problem 3

$$\begin{aligned}x'' + 2y'' - 5y' + 10y - 8x &= 0 \\4y'' + 6y' - 12x' + 2y - 14x &= 0\end{aligned}$$

Let  $u = x'$  this implies  $u' = x''$ . Let  $v = y'$  this implies  $v' = y''$ . This yields the two equations:

$$\begin{aligned}u' + 2v' - 5v + 10y - 8x &= 0 \\4v' + 6v - 12u + 2y - 14x &= 0\end{aligned}$$

Using the second equation to solve for  $v'$  yields:

$$\begin{aligned}4v' &= -6v + 12u - 2y + 14x \\2v' &= -3v + 6u - y + 7x \\v' &= \frac{-3}{2}v + 3u - \frac{1}{2}y + \frac{7}{2}x\end{aligned}$$

this implies

$$\begin{aligned}u' &= -2v' + 5v - 10y + 8x \\&= -(-3v + 6u - y + 7x) + 5v - 10y + 8x \\&= 3v + 5v - 6u - 10y + y - 7x + 8x \\&= 8v - 6u - 9y + x\end{aligned}$$

Thus putting these four equations into matrix notation we have:

$$\begin{pmatrix} x' \\ y' \\ u' \\ v' \end{pmatrix} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 1 & -9 & -6 & 8 \\ \frac{7}{2} & \frac{-1}{2} & 3 & \frac{-3}{2} \end{pmatrix} \begin{pmatrix} x \\ y \\ u \\ v \end{pmatrix}$$